Challenge Journal of Structural Mechanics

Research Article

Design of cylindrical steel liquid tanks with stepped walls using One-foot method

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ABSTRACT

Cylindrical steel tanks are used in most countries to store bulk volumes of both solid and liquid products such as water, oil, gasoline and grain. Such steel tanks are prone to buckling when subjected to external pressure either due to vacuum or due to wind. These types of shell structures are generally controlled by elastic buckling failure because of the thin wall thickness. Cylindrical shells are commonly constructed with stepwise variable wall thickness due to economic reasons. The thickness of the tank shell wall is designed to increase from top to bottom because the stress resultants on the tank wall gradually increase towards the base of the tank. For open-top tanks, a primary stiffening ring is required at or near the top to maintain roundness under all loads. Stress resultants in a primary stiffening ring were previously identified by the Author for uniform wall thick tanks. In this new study, the applicability of this hand calculation method in stepped wall tanks has been investigated. Pursuant to this goal, a specified tank shell was designed considering One-foot method. Then, the stepped wall tank was transformed into an equivalent 1-course tank for hand calculation. Using the previously developed hand calculation method by Author, a test for the in-plane bending moment in the ring was conducted to achieve an acceptable value for stepped wall tanks. The analysis results show that the previously proposed method for uniform wall thick tanks may also be used for stepped wall tanks considering an equivalent thickness. On the other hand, using Linear Buckling Analysis (LBA), the buckling mode was obtained for two different stepped wall tanks in the study.

ARTICLE INFO

Article history:
Received 20 July 2021
Revised 3 September 2021
Accepted 7 October 2021

Keywords:
Stepped wall tank
One-foot method
Primary stiffening ring
In-plane bending moment
Buckling

1. Introduction

Cylindrical steel tanks are used in most countries to store bulk volumes of both solid and liquid products such as water, oil, gasoline and grain. The thin wall makes the tank susceptible to buckling under external pressure due to wind, or partially vacuum when it is empty or partially emptied. Tank structures in the form of a cylindrical steel shell are generally controlled by elastic buckling failure because of the thin wall thickness. The behavior of the uniform wall thick tanks under environmental loads such as earthquake and wind has been explored by different research teams (Tedesco et al., 1989; Sivy and Musil, 2017; Chen and Rotter, 2012; Zdravkov, 2018; Zdravkov, 2019; Zeybek et al., 2019). However, the wall thickness of a storage tank is normally chosen to resist only the internal pressure from the stored product (Rotter et al., 2015). Instead of utilizing a uniform wall thickness, cylindrical shells of stepwise variable wall thickness are commonly constructed due to economic reasons.

As shown in Fig. 1, tank shell consists of a number of individual courses (also called strakes) each of constant thickness (Chen et al. 2011; Chen et al. 2012). The overall thickness of the tank wall is designed to increase from top to bottom because the stress resultants on the tank wall gradually increase towards the base of the tank (Chen et al., 2011, 2012).

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ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsmec.2021.04.001

Nomenclature

θ	circumferential angle measured from the windward direction
CA	corrosion allowance (mm)
C_m	coefficient for each harmonic
D	tank diameter
G	design specific gravity of the liquid to be stored
H	height of the tank or maximum design liquid level
h_i	each course height
H_i	the distance from the top of the cylinder to the bottom of the i^{th} strake
I	an integer number of strakes
I_x	second moment of area normal to the plane of the ring
L	$(500Dt)^{0.5}$
l	half-wave-height of the buckle
m	harmonic number
M_x	in-plane bending moment of ring
n	total course number
N	total number of harmonics being considered
q	wind pressure on the stagnation meridian
S_d	allowable design condition stress (MPa)
S_t	allowable hydrostatic test condition stress (MPa)
t	bottom-course corroded shell thickness (mm)
t_{1d}	design shell thickness for bottom course (mm)
t_{1t}	hydrostatic test shell thickness for bottom course (mm)
t_d	design shell thickness (mm)
t_{eq}	equivalent uniform thickness
t_i	thickness of the i^{th} strake
t_t	hydrostatic test shell thickness (mm)
V	wind speed (km/h)
Z	section modulus of the ring (cm^3)
ν	Poisson's ratio
χ	shell-ring stiffness ratio

The top of the tank can be either fixed or open-topped. Open-top tanks require a stiffening ring at or near the top to maintain roundness under all loads. For a fixed-roof tank, the stiffening ring is not needed since the tank roof provides a natural restraint. According to API 650 (2013), there are three approaches to determine the required plate thickness of the tank shell. One of them is the One-foot method which is based on shell membrane theory (Azzuni and Guzey, 2015). The method computes the required thicknesses at design points 0.3 m above the bottom of each shell course. The One-foot method is used for tanks of less than 61 m in diameter. The second approach is the Variable-design-point method which was developed by Zick and McGrath (1968). According to API 650 (2013), the applicability of Variable-design-point method is limited by Eq. (1).

$$\frac{L}{H} \le \frac{1000}{6} \tag{1}$$

The required plate thickness is determined by linear analysis for the tanks where the L/H ratio is more than 1000/6. However, API 650 (2013) does not describe a specific linear analysis method to calculate the plate thickness (Azzuni and Guzey, 2015).

The European standard for Shells EN 1993-1-6 (2007) allows transformation of stepped shell into equivalent uniform cylindrical shell considering a two-stage process. This approach based on the studies of Resinger and Greiner (1974, 1976) and Greiner (1981) includes complicated calculations that require interpolation. On the other hand, Chen et al. (2011) and Chen et al. (2012) proposed a new hand calculation method for stepped wall cylinders. Based on the research of Trahair et al. (1983), a simpler method was developed to get an equivalent uniform cylindrical shell.

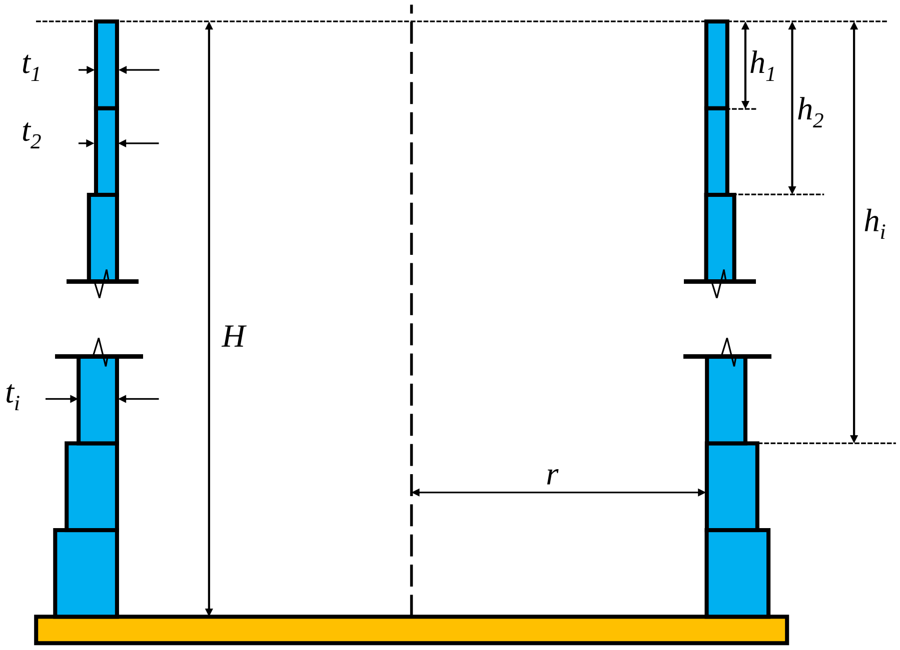


Fig. 1. Typical example of tank cross-section view (adopted from Chen et al. (2012)).

Moreover, an increase in the wall thickness is the one of the uneconomical ways to enhance the buckling resistance of the tank. To prevent this situation, a stiffening ring is placed at a suitable location to increase buckling resistance of the tanks (Rotter et al., 2015). Many researchers (Rotter et al., 2015; Bu and Qian, 2015; Bu and Qian, 2016; Dheyaaldın et al., 2017; Azzuni and Guzey, 2017; Sun et al., 2018; Zeybek et al., 2019; Zeybek and Seçer, 2020) conducted research for the sizing of such rings. For hand calculation, the step wall tank shell can be transformed into a 1-course equivalent tank shell as shown in Fig. 2.

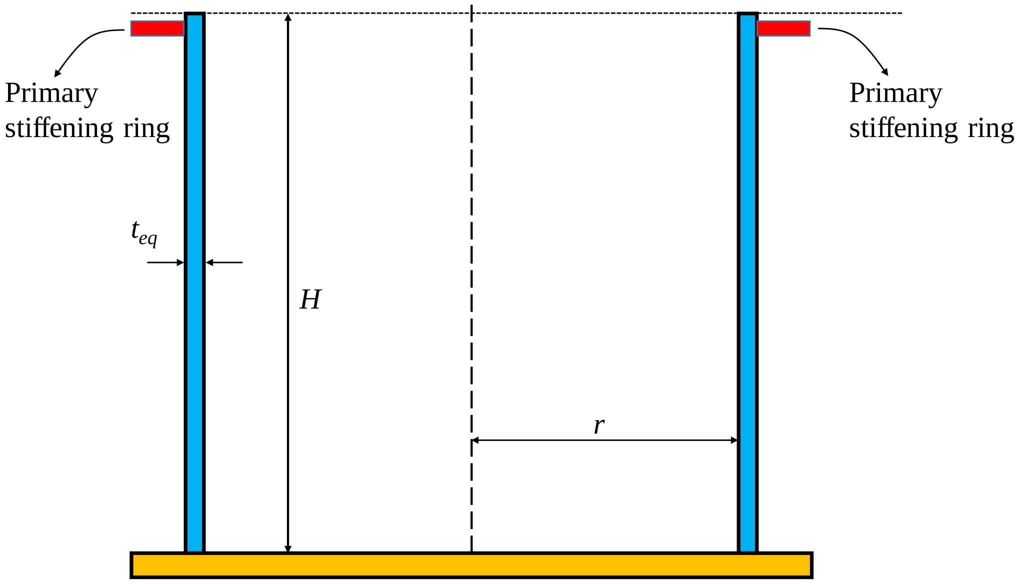


Fig. 2. Equivalent single cylinder of open-top tank with primary stiffening ring.

In this study, a tank shell wall was designed by considering the One-foot method. Then, the stepped-wall cylinder was transformed into an equivalent uniform cylindrical shell for hand calculation. After that, the size of the primary stiffening was determined by API 650 (2013) rules and in-plane bending moment variations were identified by the proposal of Zeybek et al. (2019). Moreover, the buckling mode of the specified stepped wall tanks under wind pressure was investigated by conducting Linear Bifurcation Analysis (LBA).

2. Tank Shell Wall Design Approaches

As mentioned before, three approaches have been proposed to calculate the shell thickness for storage tanks in the API 650 (2013) standard. The standard is used for the design of vertical, cylindrical, flat-bottomed and welded tanks. In this paper, these three approaches for shell design have been investigated in detail in the following sub-parts. It should be noted that below equations in the sub-parts are unit dependent. These equations in this study are in SI units.

2.1. The One-foot method

The One-foot method calculates the required plate thickness at design points 0.3 m above the bottom of each shell course. This method is valid for tanks of less than 61 m in diameter. According to API 650 (2013), the required minimum thickness of shell plates is obtained by considering the larger value calculated by the formulas below.

$$t_d = \frac{4.9\,D\,(H-0.3)\,G}{S_d} + CA \tag{2}$$

$$t_t = \frac{4.9\,D\,(H-0.3)}{S_t} \tag{3}$$

2.2. Variable-design-point method

The Variable-design-point method was proposed by Zick and McGrath (1968). It is used for tanks with the L/H ratio less than 1000/6. According to API 650 (2013), the required minimum thickness of shell plates is found by considering the larger value calculated by the formulas below.

$$t_{1d} = \left(1.06 - \frac{0.0696\,D}{H}\sqrt{\frac{H\,G}{S_d}}\right)\left(\frac{4.9\,H\,D\,G}{S_d}\right) + CA \tag{4}$$

$$t_{1t} = \left(1.06 - \frac{0.0696\,D}{H}\sqrt{\frac{H}{S_t}}\right)\left(\frac{4.9\,H\,D}{S_t}\right) \tag{5}$$

While calculating the second and upper shell courses, an iterative process is needed in the Variable-design-point method. However, there is no need for iteration in any stage of the One-foot method.

2.3. Linear analysis method

Linear analysis is used to calculate shell thickness of the tanks when One-foot method and Variable-design-point method are not permissible. It should be employed for the tanks where the L/H ratio is more than 1000/6.

However, traditional design treatments such as API 650 does not describe a specific linear analysis method to calculate the shell thickness. A recent study by Azzuni and Guzey (2015, 2016) proposed a rational analysis approach using thin shell theory. This approach also captures the plastic yielding moment of the bottom accurately when the tank exceeds limit value of 1000/6.

3. Transformation of Stepped Cylinder to Equivalent 1-Course Cylinder

The cylinder consisting of multiple sections with different wall thicknesses can be transformed into an equivalent 1-course cylinder. Chen et al. (2011) and Chen et al. (2012) developed a new hand calculation method for stepped wall cylinders. Based on the research of Trahair et al. (1983), a simple method was developed to get an equivalent uniform cylindrical shell. The new "weighted smeared wall method" was derived to find equivalent thickness. According to Fig. 1, the equivalent uniform thickness was given as follows:

$$t_{eq} = \left\{\left(\frac{1}{l}\right)\sum_{i=1}^{n}\left[t_i^3\left(H_i - H_{i-1}\right)\right]\right\}^{1/3} \quad i = 1, 2, \ldots\ldots, n \qquad (6)$$

in which $H_i = h_i - \frac{l}{2\pi}\, sin\frac{2\pi h_i}{l}$.

4. Calculation of the Primary Stiffening Ring Size

As shown in Fig. 3, there are many cross-sections for stiffening rings. These sections could be either from angle sections or formed from plates. Moreover, there are many design standards that recommend how the stiffening ring may be designed. The recommendations in the standards present a wide range of values for the ring size. There are two potential requirements (stiffness and strength) for the primary stiffening ring. One of the most common standards API 650 (2013) includes a strength design criteria for the ring. API 650 (2013) provides a section modulus (Z) according to a simplified mechanical model as follows:

$$Z = \frac{HD^2}{17}\left(\frac{V}{190}\right)^2 \qquad (7)$$

According to Eq. (7), the required minimum section modulus is directly related to design wind speed, tank height and diameter.

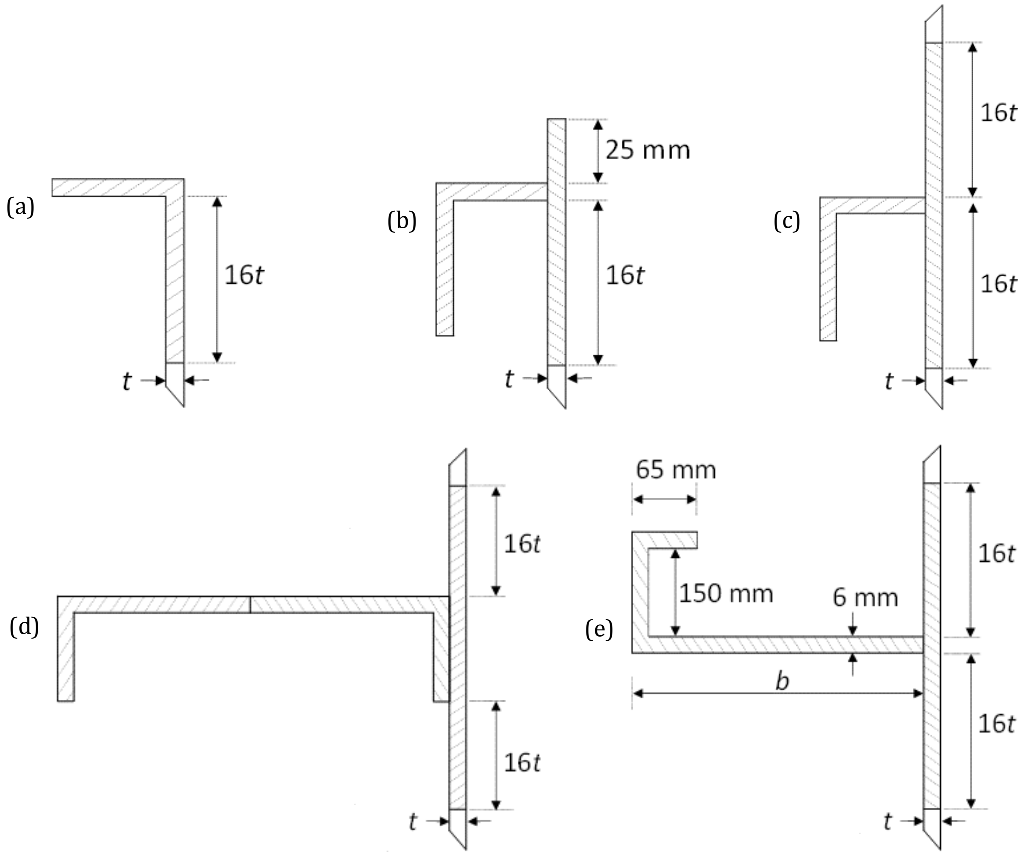


Fig. 3. Typical stiffening ring sections (adopted from API 650 (2013)):
a) Top angle; b) Curb angle; c) Single angle; d) Two angles; e) Formed plate.

A recent study by Zeybek et al. (2019) also proposed a strength criteria for the primary stiffening ring. In-plane bending moment (M_x) variation was identified considering a proposed shell-ring stiffness ratio (χ) and tributary height as follows:

$$M_x = -qr^2 H \sum_{m=2}^{N} \frac{C_m f_2(x)}{(m^2-1)} f_1(x) \cos m\theta \qquad (8)$$

where

$$f_1(x) = a_1 + \frac{1}{(1+\chi)^{a_2}} \tag{9}$$

$$f_2(x) = \frac{a_3\chi^{-a_5}}{a_4+\chi^{-a_5}} \tag{10}$$

where constants a_1, a_2, a_3, a_4 and a_5 are given in Tables 1 and 2.

Table 1. Coefficients a_1 and a_2.

H/D	a_1	a_2
1.00	0.00001	0.98
0.75	0.00002	0.98
0.50	0.00003	0.95
0.25	0.00004	0.93

Table 2. Coefficients a_3, a_4 and a_5.

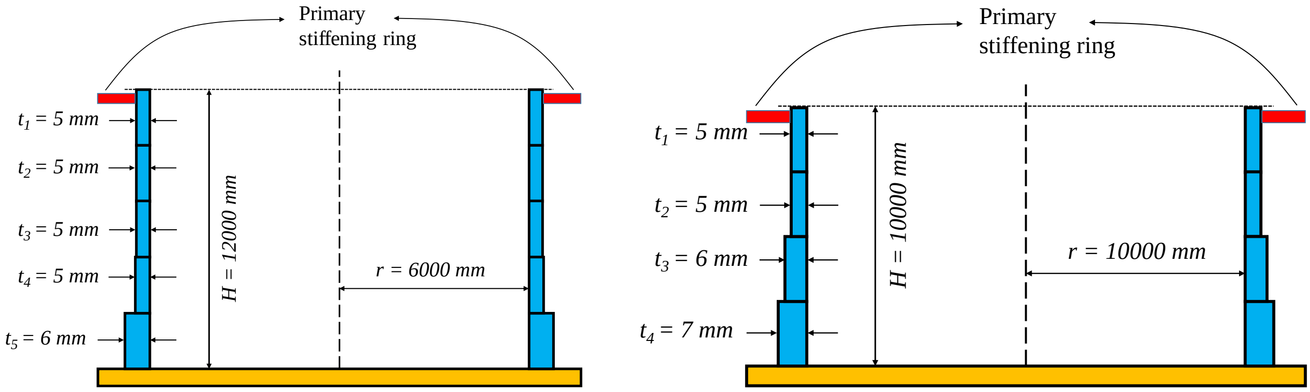
	m=2			m=3			m=4		
H/D	a_3	a_4	a_5	a_3	a_4	a_5	a_3	a_4	a_5
1.00	0.44	0.03	0.26	0.40	0.01	0.34	0.40	0.04	0.12
0.75	0.45	0.01	0.43	0.42	0.02	0.31	0.40	0.03	0.20
0.50	0.48	0.01	0.47	0.45	0.02	0.40	0.42	0.03	0.32
0.25	0.52	0.02	0.40	0.50	0.02	0.41	0.48	0.03	0.37

It should be noted above mentioned strength criteria was developed for uniform thick shells. In this study, the applicability of this approach will be investigated for stepped-wall tanks.

5. Numerical Study

Finite element analysis was used to evaluate two different tanks with stepped walls. In the first case, a tank diameter of 12 m and tank height of 12 m were selected with stepped thickness. The heights of the shell courses were 2.4 m. In the second case, a cylindrical tank with D=20 m and H=10 m was considered with stepped thickness. The heights of the shell courses were 2.5 m.

As shown in Fig. 4, thicknesses of the courses for both tank structures were computed as per API 650 (2013) using the One-foot method since the tank diameter is less than 61 m. S275 steel is used for the both tank models and the maximum allowable product design stress (S_d) was 167 MPa. The maximum allowable hydrostatic test stress (S_t) was 184 MPa as per Table 5.2a in API 650 (2013). The specific gravity of liquid was 1.0.

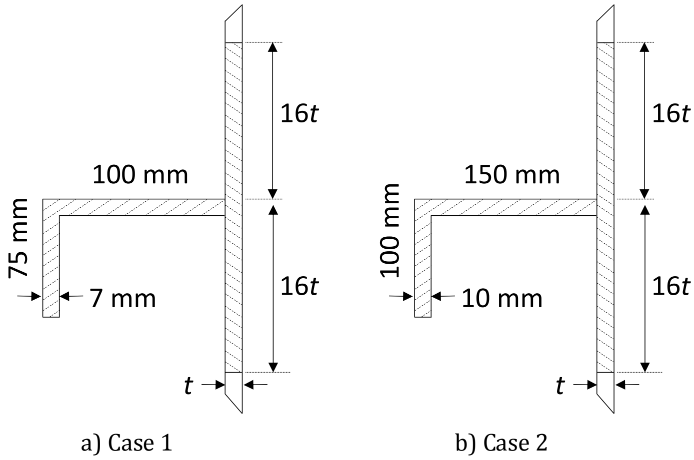


a) Case 1 b) Case 2

Fig. 4. Specific tank sections considered in numerical study.

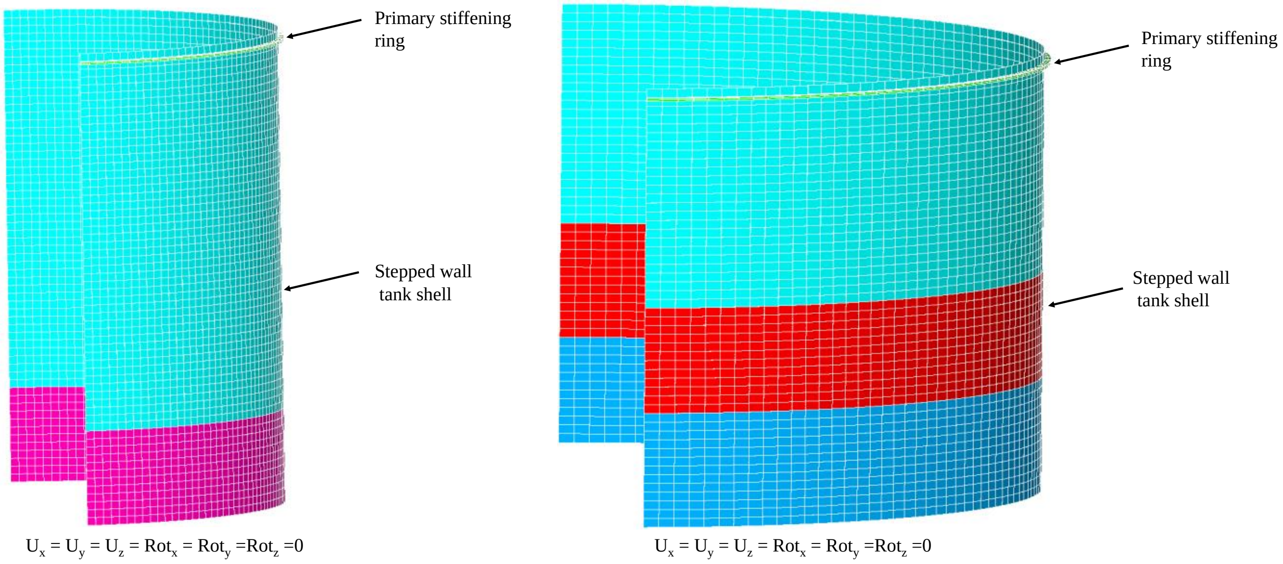
The tank structures are considered open-top, so a primary stiffening ring is required at or near the top in order to maintain circularity when they are subjected to external pressure due to vacuum or wind. In this study, a primary stiffening ring is placed 200 mm away from the top of the tanks. The primary stiffening ring for both two cases is designed in accordance with API 650 (2013) with a design wind speed of 145 km/h. The required minimum section modulus for the rings was calculated (Z_{required}=59.2 cm^3 for Case 1 and Z_{required}=137.04 cm^3 for Case 2) considering Eq. (7). Unequal angles (100x75x7 for Case 1 and 150x100x10 for Case 2) were selected. According to the Table 5.20a in API 650 (2013), the provided section modulus (Z_{provided}) for Case 1 and Case 2 were 60.59 cm^3 and 155.91 cm^3 respectively. The selected unequal angle sections are shown in Fig. 5.

The commercial finite element program ANSYS was used to perform the numerical analysis. As shown in Fig. 6, half of the model is used and symmetry boundary conditions were applied to the nodes in each symmetry plane. Four-node shell elements (shell63) were used to model the cylindrical shell. The stiffening ring was modelled using two-node beam elements (beam4). The elastic modulus was chosen as E=200 GPa and Poisson's ratio was chosen as ν=0.30. At the shell wall base, the nodes were restrained against all translational and rotational degrees of freedom (U_x=U_y=U_z=Rot$_x$=Rot$_y$ =Rot$_z$=0) to simulate a rigid bottom plate. A representative finite element mesh for two cases is shown in Fig. 6 where the line elements for the wind girder are shown in expanded form. It should be noted that wind forces act the numerical model and dynamic forces such as seismic effects were ignored in the study.



a) Case 1 b) Case 2

Fig. 5. Section of primary stiffening ring for Case 1 and Case 2.



a) Case 1 b) Case 2

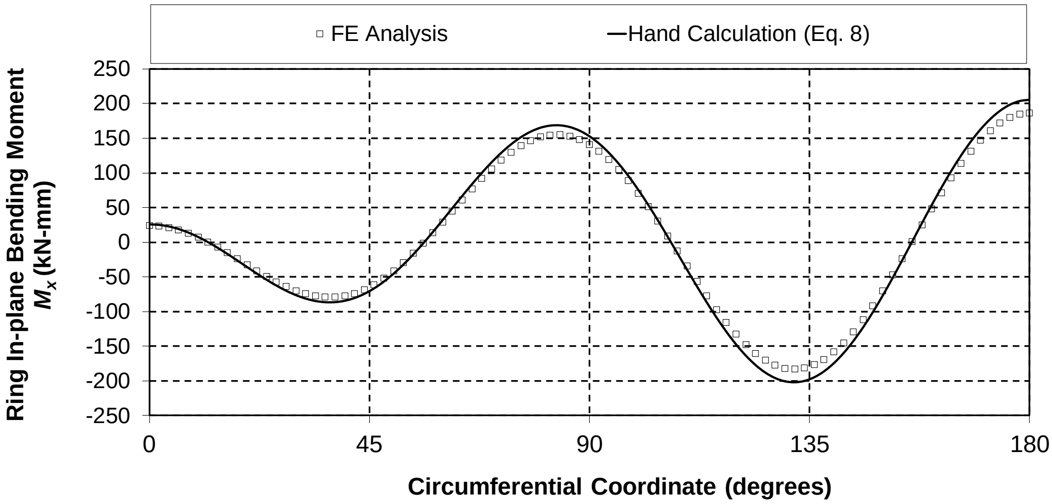
Fig. 6. FE models of the stepped wall tanks in numerical study.

Then considering the unequal angle section provided for the primary stiffening ring, the in-plane bending moment variation in the ring was calculated using Eq. (8). Since selected tank structures consist of stepped walls, an equivalent thickness is required for calculation of the in-plane bending moment. Eq. (6) was used for equivalent thickness.

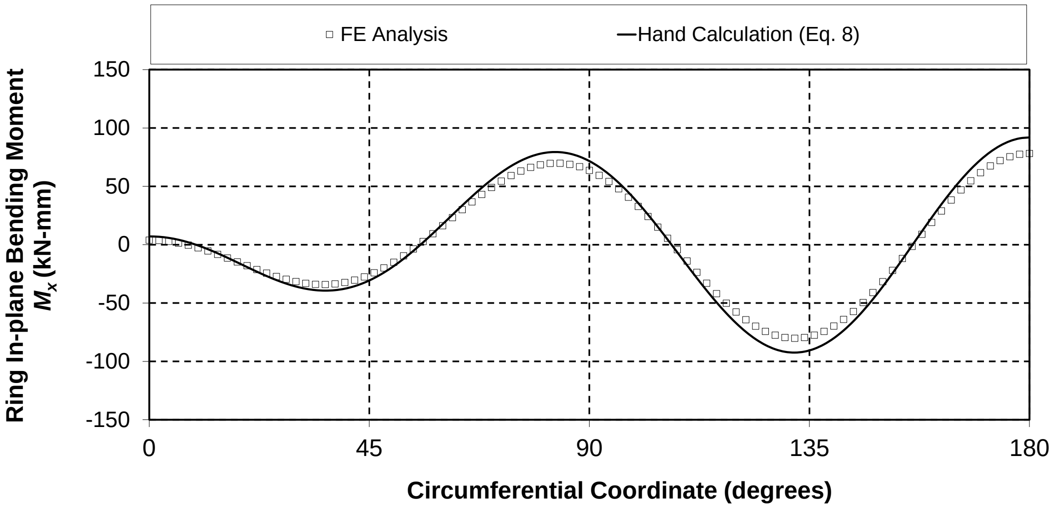
For wind pressure distributions, Greiner terms (C_0=–0.55 for m=0, C_1=0.25 for m=1, C_2=1.0 for m=2, C_3=0.45 for m=3, C_4=-0.15 for m=4) were used. The empty cylindrical tank structures were subjected to a non-uniform wind pressure in the numerical study. Both numerical and hand calculation results for in-plane bending moment variations are shown in Fig. 7.

The comparison indicates that the proposal of Zeybek et al. (2019) provides an acceptable solution for the in-plane bending moment of the ring in two different specific stepped-wall tank structures; the estimated values are on the conservative side in the study.

On the other hand, Linear Bifurcation Analysis (LBA) was performed to get buckled shape of the specified stepped wall tank. LBA determines the elastic critical buckling resistance. Hence it can be a good first estimate of the elastic buckling strength of the tank structures. The buckled shape of the stepped wall tank structures under wind pressure was shown in Fig. 8. As seen in Fig. 8, the buckle form in windward side for the specific tanks extends almost over the whole height of the tank shell and it is more pronounced in the thinnest part.

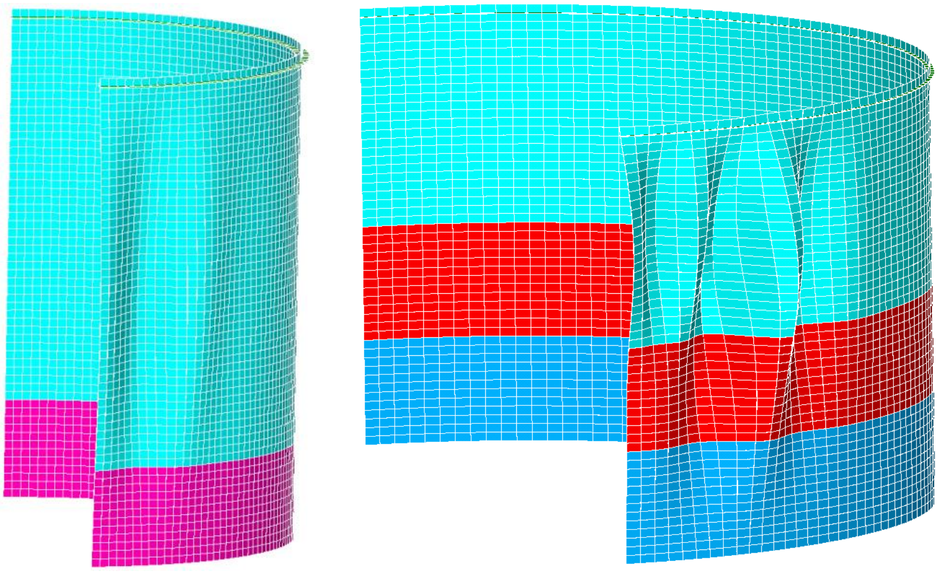


a) Case 1



b) Case 2

Fig. 7. Comparison of hand calculation with FE values for the ring in-plane bending moment (M_x).



a) Case 1 b) Case 2

Fig. 8. Buckled shapes of the stepped wall tank models in numerical study.

6. Conclusions

This study presents the design of stepped wall storage tanks with primary stiffening rings according to API 650 (2013) rules. First, two tank structures were designed considering the One-foot method, which is one of the common tank shell design approaches. Then, the required section modulus was calculated considering design wind speed, tank height and diameter. Next, in order to investigate the applicability of the proposal of Zeybek et al. (2019) for in-plane bending moment in the stepped wall tank, the stepped tank shell was transformed into a 1-course equivalent tank shell using an approach which was given in Chen et al. (2011) and Chen et al. (2012).

Analysis results of the numerical examples show that the strength criteria developed by Zeybek et al. (2019) can also be applicable for stepped wall tanks that are designed with One-foot method. Moreover, buckled shapes of the two specific tank structures were obtained by conducting LBA. The buckle form in windward side for the tanks extends almost over the whole height of the tank shell and it is more pronounced in the thinnest part.

Future studies will concentrate on design requirements for large stepped wall tanks which are designed using the Variable-design-point method. The effect of secondary stiffening rings (also called intermediate rings) on tank behavior will be also investigated.

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