Challenge Journal of Structural Mechanics

Research Article

Jaya algorithm based optimum design of reinforced concrete retaining walls under dynamic loads

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ABSTRACT

In this study, the optimum dimensioning of a reinforced concrete retaining wall that meets the safety conditions under static and dynamic loads in terms of cost has been performed using Jaya algorithm, which is one of the metaheuristic algorithms. In the optimization process, reinforced concrete design rules and ground stress, sliding and overturn tests have been determined as design constraints for the safe design of the retaining wall. While 5 cross-section dimensions of the retaining wall are defined as the design variable, the objective function is targeted as the total cost per unit length of the retaining wall. In the study, optimum results are also presented by examining the changes of the toe projection length of the retaining wall, which is one of the design variables, narrowing between 0.2-10 m. The design variables minimizing the objective function were found via Jaya algorithm that have single-phase. In addition to achieving optimum dimensioning results in terms of safety and cost with the optimization method used as a result of the reinforced concrete design made by applying the rules of the regulation on buildings to be constructed in earthquake zones, the change in cost in seismic and static conditions was examined.

ARTICLE INFO

Article history:
Received 31 August 2020
Revised 26 October 2020
Accepted 17 November 2020

Keywords:
Cantilever retaining wall
Reinforced concrete
Jaya algorithm
Optimum design
Optimization

1. Introduction

Structural design is one of the areas of expertise of civil engineering and it is the process of manufacturing buildings in a safe, economical and aesthetic way. In this process expected from engineers is to determine the type of structure suitable for the requested use and the materials to be used by calculating the loads that the building can carry. These decisions made by the engineers based on accepted rules and experiences affect many areas such as the cost of the project, its duration, the safety of the building, its applicability, etc. If the structural design variables analysed according to the predicted critical situations are insufficient or excessive, the building is redesigned by trial and error. The intensity of this process negatively affects the design process in terms of time and cost concepts that are important for engineers. Structural optimization methods provide the opportunity to reach the optimum solution by making more tests faster than an engineer can do by hand.

Structural design engineers construct retaining walls with rigid supporting structures in order to hold the ground volume at two different levels. Engineers must calculate various loads such as active thrust behind the wall to which the retaining wall will be exposed, passive thrust in front of the wall, surcharge loads and earthquake effects during the design process. Incomplete calculation of these loads affecting the retaining walls may affect the stability of the wall negatively, and the cost of the retaining wall is not economical in unnecessary loads. The pre-dimensions of the reinforced concrete cantilever retaining walls, which are one of the rigid retaining walls, are tried to be made by benefiting from the accepted knowledge and the experience of the engineer. First of all, the initial values of the other cross-section lengths of the wall are determined by making some assumptions in certain regions by correlating with the wall height (Clayton, 2014). By calculating the static and dynamic loads that will affect the wall of the obtained size, a process consisting of many steps such overturning,

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ISSN: 2149-8024 / DOI: https://doi.org/10.20528/cjsmec.2021.02.002

sliding and stress stability analysis of the wall and in addition to these investigations, reinforcement areas changing according to internal forces, reinforcement ratios not exceeding the limit values are performed. If the controls result is negative, the process cycle is repeated by pre-sizing according to the accepted ratios. This cycle is not a practical method, as testing all design possibilities and finding the best solution will be both a time consuming and tedious process. Structural optimization methods are used because of these difficulties.

Optimum design studies of reinforced concrete cantilever retaining walls started in the 1980s, and after the 2000s, the focus was on the applicability of metaheuristic algorithms in optimization. The optimum design of the reinforced concrete retaining wall under static loads, using Simulated Annealing (SA) algorithm inspired by the annealing of metals, has been studied on the parameters that affect the minimum cost (Yepes et al., 2008) and economic design (Ceranic et al., 2001). The optimum design of the reinforced concrete retaining wall under static and dynamic loads was made by the methods of Collision Bodies Optimization (CBO) and Democratic Particle Swarm Optimization (DPSO) (Kaveh and Soleimani, 2015). As an example of metaheuristic algorithms used in optimum design of reinforced concrete retaining wall; Particle Swarm Optimization(PSO) inspired by the search for food by flocks of birds and fish (Ahmadi and Varaee, 2009) , Ant Colony Optimization (ACO), which is found by making use of the destination ants follow on their way to food (Ghazavi and Bonab, 2011), Harmony Search (HS) Algorithm inspired by the harmony between music rhythms (Kaveh and Abadi, 2011), Big Bang-Big Crunch (BP-BC) inspired by the cosmological model in the theory of evolution of the universe (Camp and Akin, 2012), Genetic Algorithm (GA) that utilizes changes in gene sequence (Pei and Xia, 2012), Firefly Algorithm (FA) developed on the basis of brightness-sensitive social behaviors of fireflies (Akin and Aydogdu, 2014), Teaching-Learning Based Optimization Algorithm (TLBO) inspired by a teacher's effect on students in the classroom (Rao et al., 2011), and Jaya Algorithm, which aims to achieve victory by moving away from the bad solution and approaching the good solution (Rao, 2016) can be shown. The optimum design of the reinforced concrete retaining wall under both static and dynamic loads was made by Temür and Bekdaş (2016) by applying the Teaching-Learning Based Optimization Algorithm (TLBO). There is a study using the Jaya Algorithm to design an optimum retaining wall, but in the mentioned study, the effect of the size of the surcharge load on the cost of the wall and the CO_2 emission value are examined in the mentioned study (Ozturk and Turkeli, 2019). In the literature, optimum design of reinforced concrete retaining wall under earthquake loads additional to static loads has not been encountered with Jaya algorithm.

In this study, it is aimed to design a reinforced concrete cantilever retaining wall designed in accordance with the legislation on Buildings to be built in Earthquake Zones (DBYBHY) (Ministry of Public Works and Settlement, 2007) by using Jaya algorithm, which is one of the metaheuristic algorithms, at optimum cost under static and dynamic loads. In addition, two different designs obtained for earthquake-free and earthquake situations were taken as reference and the relationship between earthquake and cost was examined. For the earthquake state design, the toe projection length limit values of the reinforced concrete retaining wall were also changed, and it was seen that the cost increased as the toe projection length limits were reduced by comparing 6 cases.

2. Design Methodology

2.1. Optimization problem

Retaining walls are exposed to vertical and horizontal loads shown in Fig. 1. External forces acting under static conditions (such as the mass of the wall (G_p, G_{on}, and G_t), soil pressures (P_{as}, Q_{as}) maintain their balance in the rest state. On the other hand, dynamic forces (P_{ad}, Q_{ad}) that occur during an earthquake can disrupt the balance and consequently cause permanent deformations in the wall. In case of extreme deformations, sliding, rotation, bending, etc. crash occurs for reasons. Retaining walls can be safely designed by performing a soil stress check on the base, a forward sliding check along the base and an overturning test at the heel of the wall to prevent collapse.

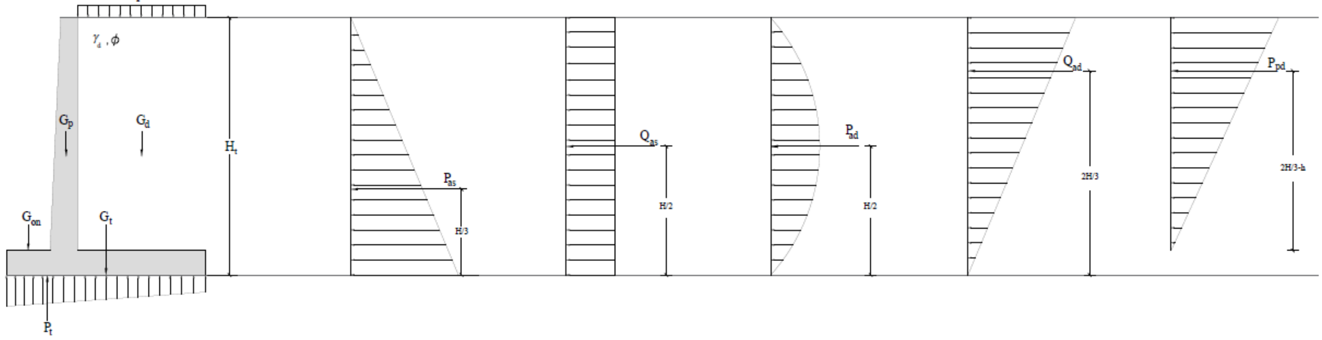

Fig. 1. Static and dynamic loads that can affect a cantilever retaining wall.

For ground stress analysis, the maximum and minimum soil stresses under the base are calculated as shown in Eq. (1). In this equation, N is the axial force, the total area A_t, the moments occurring with respect to the base midpoint M_o and the moment of resistance W_t. The largest ground stress (σ_{max}) in the base must be smaller than the ground safety stress (σ_u) (Eq. (2)). The smallest ground stress ($\sigma_{\min}$) on the base should be designed to be positive to avoid undesired tensile stresses (Eq. (3)). In case of earthquake loading, the ground safety stress can be increased by 50% due to the earthquake acting for a limited time (Eq. (4)).

$$\sigma_{\max,\min} = \frac{N}{A_t} \pm \frac{M_O}{W_t} \tag{1}$$

$$\sigma_{\max} < \sigma_u \tag{2}$$

$$\sigma_{\min} > 0 \tag{3}$$

$$\sigma_{\max} < 1.5\, \sigma_u \tag{4}$$

The impulse from the embankment and surcharge load forces the base of the retaining wall to slide. The sliding test of the retaining wall is expressed as the ratio of the forces (F_R) against the sliding to the forces (F_O) that cause the wall to slide (Eq. (5)). The safety number (SF_S) has been accepted as at least 1.5 in granular soils and at least 1.1 under earthquake loads (Yıldırım, 2009). If the safety number is not sufficient, the safe state can be reached more economically by increasing the base width.

$$SF_S = \frac{\sum F_R}{\sum F_O} \tag{5}$$

The loads generated behind the retaining wall tend to tip the wall. The overturning safety coefficient (SF_O) is expressed as the ratio of the moments taken relative to the lower end of the toe projection to the forces (M_O) trying to overturn the system (M_R) (Eq. (6)). SF_O should only meet the condition of at least 2 under static loads and at least 1.2 under static and dynamic loads. If these conditions are not met, overturning safety can be provided by increasing the moment arm of the M_R by extending the heel projection.

$$SF_O = \frac{\sum M_R}{\sum M_O} \tag{6}$$

In the optimization problem examined in this study, five design variables were used, namely stem thickness at the top of the wall ($b1$), toe projection length ($b2$), stem thickness at the bottom of the wall ($b3$), heel projection length ($b4$) and base slab thickness (h) (Fig. 2). In the calculation of these variables, Eqs. (2-6) are considered as design constraints.

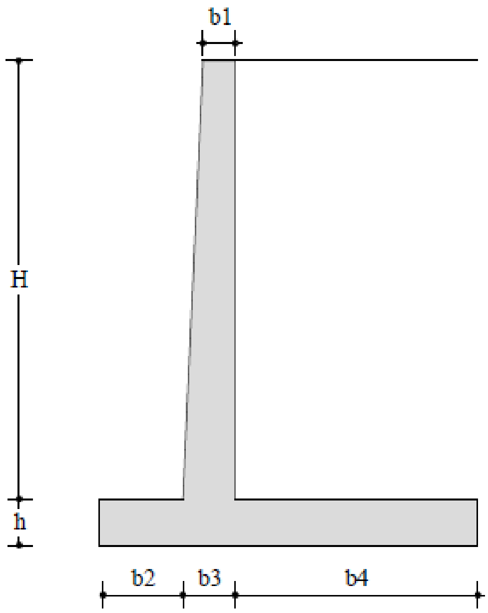

Fig. 2. Design variables of a cantilever retaining wall.

Eqs. (2-6) mentioned above are determined as design constraints. Preliminary dimensions, which are accepted in height-dependent proportions and certain limit values in cantilever reinforced concrete retaining walls, are also included in the optimization problem as a design constraint (Clayton, 2014). In addition, critical internal force (shear and bending) in the cantilever, toe and heel projection for non-earthquake and earthquake conditions were calculated and the minimum and maximum reinforcement areas were checked. Table 1 presents design restrictions.

Table 1. Restrictions on the strength of the retaining wall.

Explanation	Restricts	
	Under static loads	Under static and Dynamics loads
Safety for bearing capacity	$\sigma_{\max,\text{design}} < \sigma_u$ $\sigma_{\min,\text{design}} > 0$	$\sigma_{\max,\text{design}} < 1.5\,\sigma_u$ $\sigma_{\min,\text{design}} > 0$
Safety for sliding stability	$SF_{S,\text{design}} > 1.5$	$SF_{S,\text{design}} > 1.1$
Safety for overturning stability	$SF_{O,\text{design}} > 1.5$	$SF_{O,\text{design}} > 1.2$
Maximum reinforcement area of console section, $A_{s,\max}$	$A_{s,\text{design}} < A_{s,\max}$	$A_{sd,\text{design}} < A_{s,\max}$
Minimum reinforcement area of console section, $A_{s,\min}$	$A_{s,\text{design}} \geq A_{s,\min}$	$A_{sd,\text{design}} \geq A_{s,\min}$
Maximum reinforcement area of toe projection, $A_{so,\max}$	$A_{so,\text{design}} < A_{so,\max}$	$A_{sod,\text{design}} < A_{so,\max}$
Minimum reinforcement area of toe projection, $A_{so,\min}$	$A_{so,\text{design}} \geq A_{so,\min}$	$A_{sod,\text{design}} \geq A_{so,\min}$
Maximum reinforcement area of heel projection, $A_{sa,\max}$	$A_{sa,\text{design}} < A_{sa,\max}$	$A_{sad,\text{design}} < A_{sa,\max}$
Minimum reinforcement area of heel projection, $A_{sa,\min}$	$A_{sa,\text{design}} \geq A_{sa,\min}$	$A_{sad,\text{design}} \geq A_{sa,\min}$
Shear strength capacities ocritical sections, $V_{\max}$	$V_{\max,\text{design}} \leq V_{cr}/2$	$V_{\max,\text{design}} \leq V_{cr}/2$

Cost minimization is aimed in optimum design. In this direction, the mathematical representation of the objective function is as in Eq. (7). C_c represents the unit concrete cost, V_c represents the unit length concrete volume, C_s represents the unit reinforcement cost, W_s represents the unit length reinforcement steel weight. While C_c and C_s are independent of optimization, V_c and W_s values vary according to design variables.

$$\min f(x) = C_c \cdot V_c + C_s \cdot W_s \tag{7}$$

2.2. Optimization process

There are many types of metaheuristic algorithms used in engineering applications in the literature. Jaya algorithm, one of these metaheuristic algorithms, was developed by Rao (2016). Jaya, a word that means victory in Sanskrit, is an algorithm that aims to achieve victory by reaching the best solution thanks to the goal function obtained by multiplying the design variables with random coefficients, constantly moving away from bad solutions and approaching good solutions.

First of all, the objective function ($f(x)$) should be determined for the targeted maximization or minimization problem as in other algorithms. Population number, design variables and maximum number of iterations are essential parameters for the Jaya algorithm. Randomly generated design variables within the range of design constraints are recorded in the initial solution matrix. The size of the solution matrix is the population number determined by the designer.

The design variables produced in the initial solution matrix are reused in the iterative process using the Jaya algorithm and stored as the best solution ($f(x)_{\text{best}}$) and worst solution ($f(x)_{\text{worst}}$) values. New design variables are obtained by summing the best solution difference and subtracting the worst solution difference. The formulation of this expression is shown in Eq. (8) as the updated value of the jth variable of the kth population. Candidate solution in the ith iteration is shown as $X'_{i,k,i}$.

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i}\left(X_{j,best,i} - |X_{j,k,i}|\right) - r_{2,j,i}\left(X_{j,worst,i} - |X_{j,k,i}|\right) \tag{8}$$

Since $r_{1,j,i}$ and $r_{2,j,i}$ are random numbers in the range of [0, 1] in the above equation, they ensure that new different solutions are obtained continuously. While $X_{j,k,i}$ is the value of the previous design variable, $X_{j,\text{best},i}$ shows the design variable values in the best solution so far and $X_{j,\text{worst},i}$ in the worst solution. If the updated variable $X'_{j,k,i}$ gives a better objective function, $X_{j,k,i}$ is replaced by $X'_{j,k,i}$. After checking for compliance with design constraints, it is recorded in the optimum solution matrix and used as input data in the next iteration.

This process is repeated until the specified stop condition is met. If the termination criteria are met, optimization is complete. The flow diagram of the Jaya algorithm is presented in Fig. 3.

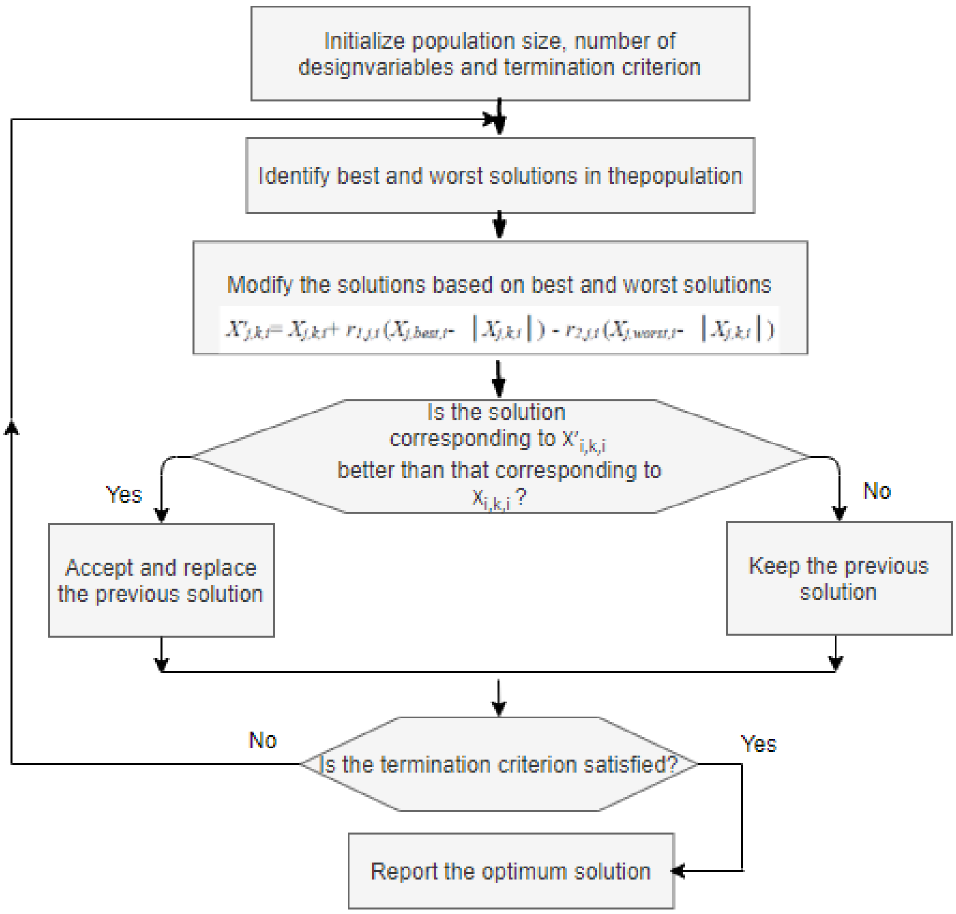

Fig. 1. Jaya algorithm flow chart (Rao, 2016).

3. Numerical Examples

The methodology has been studied for two different cases depending on static and dynamic loads. In Table 2, the design of the cantilever retaining wall (Solution 1) made by Celep (2014) was optimized using the Jaya algorithm in the first case only under static loads and named as the Solution 2. In the second case, in addition to static loads, the cantilever retaining wall was dimensioned under dynamic loads and the results were expressed as the Solution 3. The results of the design of Celep (2014) and the optimization studies made with the Jaya algorithm were compared.

Calculations are made for 1 m length of cantilever retaining wall. As the design parameters of the cantilever retaining wall in the second degree earthquake zone; the internal friction angle of the embankment is 30°, and the unit volume weight is 18 kN/m², the soil safety stress is 200 kN/m², the material class is C30/S420, the surcharge load is 5 kN/m² and the wall height (H) is 5.4 m excluding the floor height (h). The unit price of steel is 1400 TL/ton and the unit price of concrete is 111 TL/m³.

The design variables are as shown in Fig. 2. The objective function changes according to the above design constants and design variables as in Eq. (7). The stop criterion (maximum number of iterations) for optimization is 100000 and the population number is 20. All dimensions, reinforcements and total costs are given in Tables 2-4, respectively.

The optimization process of Jaya Algorithm based methodology was repeated for 10 times and average results are presented in Table 5 with the standard deviation results. Since the standard deviation results are zero, the proposed methodology is robust.

Table 2. Geometric size comparison.

Variables (m)	Range (m)	Solution 1	Solution 2	Solution 3
h	0.2-3.0	0.6000	0.4480	0.4536
$b1$	0.2-3.0	0.2500	0.2000	0.2000
$b2$	0.2-10	0.9000	1.2303	1.3076
$b3$	0.2-3.0	0.6000	0.6993	0.7123
$b4$	0.2-10	3.0000	1.9760	2.1296

Table 3. Reinforcement area comparisons on console (A_s), toe projection ($A_{s,o}$) and heel projection($A_{s,a}$).

Reinforcement areas (mm²)	Solution 1	Solution 2	Solution 3
A_s	1578.6	1278	1304
$A_{s,o}$	1080.0	776.0	787.3
$A_{s,a}$	1298.9	1156.5	1146

Table 4. Total cost comparison.

	Solution 1	Solution 2	Solution 3
Total Cost (TL/m)	2286.9	1789.7	1854.0

Table 5. Comparison of 10 runs solved by Jaya algorithm with earthquake and without earthquake cases.

Cases		h (m)	$b1$ (m)	$b2$ (m)	$b3$ (m)	$b4$ (m)	A_s (m²)	$A_{s,o}$ (m²)	$A_{s,a}$ (m²)	$f(x)$ Cost (TL/m)
Without Earthquake	Avg.	0.448	0.2	1.230	0.699	1.976	1278	776	1156	1789.7
	Std.	0	0	0.0004	0.0001	0	0.0001	0.0005	0,0003	0
With Earthquake	Avg.	0.4536	0.2	1.307	0.713	2.129	1305	787.3	1146	1854
	Std.	0.0001	0	0.0004	0	0	0	0.0001	0.0003	0

In addition, the effect of toe-projection design limit change on the cost function was checked for 6 cases. In Table 6, the effect of the amendment of the toe-projection design limit on the retaining wall section dimensions; they are compared on the basis of best, average and standard deviation values.

4. Conclusions

In this study, the optimum design of the cantilever reinforced concrete retaining wall in an earthquake-free and earthquake condition has been made using the Jaya algorithm according to DBYBHY (2007) rules. In order to test the performance and success of the method, analyses aimed at optimum cost under design constraints using the Jaya algorithm were performed and compared with a retaining wall problem solved using the same design constants (Celep, 2014).

When the design cost under static loads presented by Celep (2014) was calculated according to the unit costs used in this study, 2255 TL/m was obtained. It has been observed that the design cost in non-earthquake condition obtained by the optimization method used in the study is approximately 21% (465.3 TL/m) more economical. In addition, the optimization of the cantilever reinforced concrete retaining wall was made under both static loads and dynamic loads. The design under static and dynamic loads presented by Celep (2014) was calculated as 2286.9 TL/m according to the unit costs used in this study. In this case, it was seen that the design in earthquake condition obtained by optimization was approximately 19% (432.9 TL/m) more economical. These analyzes have shown that the optimization made with the Jaya algorithm gives more efficient results.

When the analysis performed for 6 different design intervals in the toe-projection was compared, it was determined that the standard deviation increased when the design range was narrowed, and it was not robust comparing to other cases according to the multiple cycles of the optimization process.

In the study, the analysis of the reinforced concrete retaining wall under seismic and earthquake conditions was made and optimum design results were obtained. When the comparison is made according to these results, it is seen that the analysis results without dynamic loads are approximately 4% (64.9 TL/m) lower cost as predicted. Since the load will increase with the addition of dynamic loads to the static loads, it has been determined that the wall dimensions and the required reinforcement areas should be increased by approximately 1.0259. Therefore, it was understood that dynamic impulses caused the cost of reinforced concrete retaining wall to increase. Thanks to the study, the reinforced concrete section dimensions that provide the economic and safety constraints of the reinforced concrete cantilever retaining wall were calculated with the Jaya algorithm and a more efficient design was made by examining the relationship between the earthquake load and the cost of the retaining wall.

Table 6. The Effect of pre-encasement design range on optimization.

Variables	Case	$b2$ limits	$f(x)$ $*10^3$	$b1$ (m)	$b2$ (m)	$b3$ (m)	$b4$ (m)	h (m)	A_s $*10^3$	$A_{s,a}$ $*10^3$	$A_{s,o}$ $*10^3$
best			1.854	0.200	1.307	0.713	2.129	0.454	1.305	1.146	0.787
average	1	0.2-10	1.854	0.200	1.307	0.712	2.130	0.454	1.305	1.147	0.787
std. deviation			0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
best			1.854	0.200	1.306	0.712	2.130	0.453	1.305	1.148	0.787
average	2	0.2-8	1.854	0.200	1.307	0.712	2.130	0.454	1.305	1.147	0.787
std. deviation			0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000
best			1.854	0.200	1.306	0.712	2.130	0.453	1.305	1.148	0.787
average	3	0.2-6	1.919	0.200	1.265	0.687	2.192	0.464	1.387	1.178	0.807
std. deviation			0.206	0.000	0.134	0.079	0.196	0.032	0.261	0.098	0.064
best			1.854	0.200	1.306	0.712	2.130	0.454	1.305	1.147	0.787
average	4	0.2-4	1.946	0.200	1.289	0.678	2.108	0.450	1.473	1.187	0.798
std. deviation			0.291	0.000	0.056	0.110	0.070	0.011	0.533	0.125	0.036
best			1854	0.200	1.307	0.712	2.130	0.454	1.305	1.146	0.787
average	5	0.2-2	1.930	0.239	1.377	0.700	2.092	0.481	1.345	1.166	0.843
std. deviation			0.240	0.122	0.219	0.040	0.120	0.088	0.128	0.062	0.176
best			2.792	0.200	0.000	0.712	3.477	0.754	1.304	2.795	0.000
average	6	0	2.862	0.200	0.000	0.728	3.488	0.729	1.335	2.897	0.000
std. deviation			0.221	0.000	0.000	0.050	0.036	0.078	0.100	0.321	0.000

Publication Note

This research has previously been presented at the 6th International Conference on Harmony Search, Soft Computing and Applications (ICHSA 2020) held in İstanbul, Turkey, on July 16-17, 2020. Extended version of the research has been submitted to Challenge Journal of Structural Mechanics and has been peer-reviewed prior to the publication.

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