



## Research Article

# Evaluation of artificial neural network-based formulations for tuned mass dampers

Melda Yucel <sup>a,\*</sup> , Sinan Melih Nigdeli <sup>a</sup> , Gebrail Bekdaş <sup>a</sup> 

<sup>a</sup> Department of Civil Engineering, İstanbul University-Cerrahpaşa, 34320 İstanbul, Turkey

## ABSTRACT

Tuned mass dampers (TMDs) are used to damp vibration of mechanical systems. TMDs are also used on structures to reduce the effects of strong forces such as winds and earthquakes. For the efficiency of TMD, optimization of TMD parameters is needed. Several classical formulations were proposed, but metaheuristic methods are generally used to find the best result. In addition, the metaheuristic based optimum results are used in machine learning of artificial intelligence-based models like artificial neural networks (ANN). These ANN models are also used in development of tuning equation via curve fitting. The classical and ANN-based formulations were found according to frequency domain responses. In the present study, the classical and ANN-based formulations were evaluated by comparing on time-history responses of seismic structure. In comparison, near-fault ground motion records including directivity pulses are used. The ANN based methods have advantages by providing smaller stroke requirement and damping for TMDs.

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## 1. Introduction

In order to reduce structural vibrations, several control methods have been proposed. These methods may be passive, active, semi-active or hybrid control methods. All systems of different control strategies need to be optimized to find a perfect tuning of parameters according to controlled structure subjected to vibration sources like earthquakes, winds and traffics.

Tuned mass dampers are passive vibration control devices which are used in general mechanical systems. For this purpose, several simplified equations are proposed (Den Hartog, 1947; Warburton, 1982; Sadek et al., 1997; Leung and Zhang, 2009; Yucel et al., 2019). These equations were derived according to mathematical equations, curve fitting of numerical iteration and optimum results as details given in Section 2.1.

In recent years, the most chosen methods for optimum tuning of mass dampers is the usage of metaheuristic methods. As examples, the usage of harmony search (HS) (Bekdaş and Nigdeli, 2011; Nigdeli and Bekdaş, 2017), particle swarm optimization (Khatibinia et al.,

2016), ant colony algorithm (Viana et al., 2008), genetic algorithm (Mohebbi et al., 2013; Pourzeynali et al., 2013; Arfiadi, 2016), artificial bee colony optimization (Farshidianfar and Soheili, 2013), simulated annealing (Yang and Li, 2016), cuckoo search (Etedali and Mollayi, 2018), gravitational algorithm (Khatibinia et al., 2018), bat algorithm (Bekdaş et al., 2018) and flower pollination (FPA) (Bekdaş et al., 2017); besides the usage of both harmony search, flower pollination algorithm, teaching-learning based optimization and jaya algorithm (Bekdaş et al., 2019) were proposed to optimize different types of TMDs for different objectives about responses of structures.

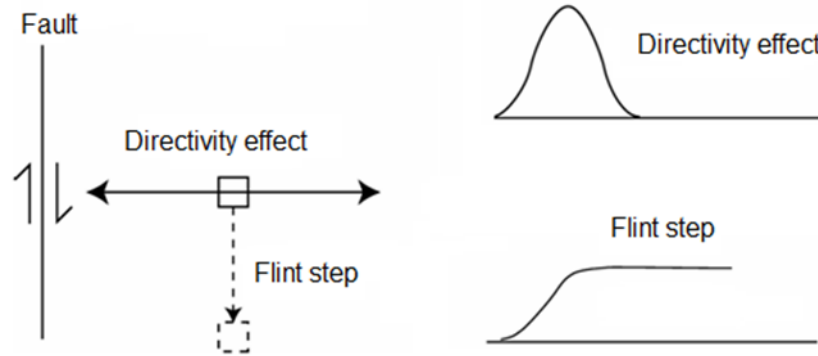
The process of numerical optimization method may need high amount of computation time. For that reason, Yucel et al. (2019) used the optimum TMD parameters on machine learning of an artificial neural network (ANN) model, and several equations were developed for easy use of engineers.

The classical and ANN-based formulations were found according to frequency-domain response of structures. The exact effect of TMDs can be only seen on time-history responses. For that reason, several formulations

\* Corresponding author. E-mail address: melda.yucel@yahoo.com.tr (M. Yucel)

are investigated on time-history responses of seismic structures by using near-fault ground motions. As seen

in Fig. 1, near fault ground motion have two significant impulsive motions.



**Fig. 1.** Impulsive motions.

Directivity pulses, which are critically effective on structures, occur in the direction perpendicular to fault (Steward et al., 2001). These pulses have long period and high peak ground velocity. In FEMA P-695: Quantification

of Building Seismic Performance Factors (2009), several near fault ground motion records with pulses are grouped, and these records given in Table 1 were used in the present study.

**Table 1.** Earthquake set for near-field excitations with pulses (FEMA P-695, 2009).

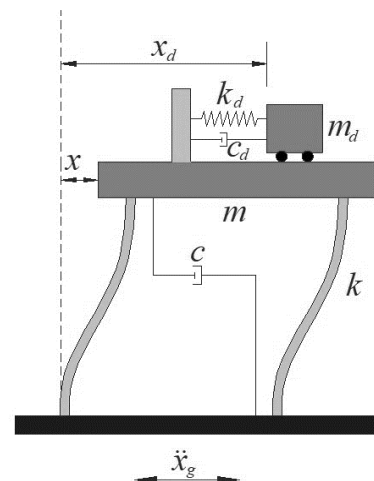
| Earthquake No. | Earthquake Name       | Recording Station        | Year | Magnitude |
|----------------|-----------------------|--------------------------|------|-----------|
| 1              | Imperial Valley-06    | El Centro Array #6       | 1979 | 6.5       |
| 2              | Imperial Valley-06    | El Centro Array #7       | 1979 | 6.5       |
| 3              | Irpinia, Italy-01     | Sturno                   | 1980 | 6.9       |
| 4              | Superstition Hills-02 | Parachute Test Site      | 1987 | 6.5       |
| 5              | Loma Prieta           | Saratoga – Aloha         | 1989 | 6.9       |
| 6              | Erzican, Turkey       | Erzican                  | 1992 | 6.7       |
| 7              | Cape Mendocino        | Petrolia                 | 1992 | 7.0       |
| 8              | Landers               | Lucerne                  | 1992 | 7.3       |
| 9              | Northridge-01         | 01 Rinaldi Receiving Sta | 1994 | 6.7       |
| 10             | Northridge-01         | 01 Sylmar - Olive View   | 1994 | 6.7       |
| 11             | Kocaeli, Turkey       | Izmit                    | 1999 | 7.5       |
| 12             | Chi-Chi, Taiwan       | TCU065                   | 1999 | 7.6       |
| 13             | Chi-Chi, Taiwan       | TCU102                   | 1999 | 7.6       |
| 14             | Duzce, Turkey         | Duzce                    | 1999 | 7.1       |

## 2. Design of Tuned Mass Dampers

### 2.1. Equations of motion

In Fig. 2, a TMD attached to a single degree of freedom (SDOF) structures is shown.  $\ddot{x}_g$  is the recorded acceleration of ground motions. The response of the structure is the displacement ( $x$ ). The structural parameters taken as the design constants are the mass ( $m$ ), stiffness ( $k$ ) and the damping coefficient ( $c$ ), which are found according to the inherent damping ( $\xi$ ) of the structure as given in Eq. (1).

$$c = 2m\xi\sqrt{\frac{k}{m}} \quad (1)$$



**Fig. 2.** SDOF structure with TMD.

The parameters of TMD are mass ( $m_d$ ), stiffness ( $k_d$ ) and damping coefficient ( $c_d$ ) of the structure. Generally, the mass of TMD is optimum at the highest allowed value according to axial loading capacity of the structure. In tuning formulations, mass ratio ( $\mu$ ) is defined as given in Eq. (2).

$$\mu = \frac{m_d}{m} \quad (2)$$

In matrix form, the equation of motion of structure coupled with a TMD is given as Eq. (3). The mass, damping and stiffness matrices are shown as  $M$ ,  $C$  and  $K$ , respectively. These matrices and the displacement vector ( $x(t)$ ) are shown in Eqs. (4-7).

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M[1]\ddot{x}_g(t) \quad (3)$$

$$M = \begin{bmatrix} m & 0 \\ 0 & m_d \end{bmatrix} \quad (4)$$

$$C = \begin{bmatrix} c + c_d & -c_d \\ -c_d & c_d \end{bmatrix} \quad (5)$$

$$K = \begin{bmatrix} k + k_d & -k_d \\ -k_d & k_d \end{bmatrix} \quad (6)$$

$$x(t) = \begin{Bmatrix} x \\ x_d \end{Bmatrix} \quad (7)$$

The dots on  $x(t)$  represent a derivative of  $x(t)$  respect to time. Solving of  $x$  needs long calculation time, and the results are specific for the excited ground acceleration. Due to that, the response can be investigated in frequency domain ( $\omega$ ) by calculating the amplitude ( $f$ ) of transfer function ( $TF(\omega)$ ).  $TF(\omega)$  is the ratio of Laplace transformations of acceleration of the system and ground.  $TF(\omega)$  and  $f$  are formulated as Eqs. (8) and (9), respectively.

$$TF(\omega) = \begin{bmatrix} TF \\ TF_d \end{bmatrix} = [-M\omega^2 + C\omega j + K]^{-1} M\omega^2 \{1\} \quad (8)$$

$$f = 20\text{Log}_{10}|\max(TF)| \quad (9)$$

## 2.2. Tuning equations for TMDs

In Table 2, the compared tuning equations with ANN-based equations are given. The equations are for optimum frequency ratio ( $f_{opt}$ ) and optimum damping ratio TMD ( $\xi_{d,opt}$ ).  $f_{opt}$  and  $\xi_{d,opt}$  are formulated as Eqs. (10) and (11), respectively.

**Table 2.** The frequency and damping ratio expressions of the compared methods.

| Method                 | $f_{opt}$   | $\xi_{d,opt}$  |
|------------------------|---|--|
| Den Hartog (1947)      | $\frac{1}{1+\mu}$   | $\sqrt{\frac{3\mu}{8(1+\mu)}}$   |
| Warburton (1982)       | $\frac{\sqrt{1 - (\frac{\mu}{2})}}{1+\mu}$  | $\sqrt{\frac{\mu(1 - \frac{\mu}{4})}{4(1+\mu)(1 - \frac{\mu}{2})}}$                  |
| Sadek et al. (1997)    | $\frac{1}{1+\mu} \left[ 1 - \xi \sqrt{\frac{\mu}{1+\mu}} \right]$   | $\frac{\xi}{1+\mu} + \sqrt{\frac{\mu}{1+\mu}}$                                       |
| Leung and Zhang (2009) | $\frac{\sqrt{1 - (\frac{\mu}{2})}}{1+\mu} + (-4.9453 + 20.2319\sqrt{\mu} - 37.9419\mu)\sqrt{\mu}\xi + (-4.8287 + 25.0000\sqrt{\mu})\sqrt{\mu}\xi^2$ | $\sqrt{\frac{\mu(1 - \frac{\mu}{4})}{4(1+\mu)(1 - \frac{\mu}{2})}} - 5.3024\xi^2\mu$ |

$$f_{opt} = \frac{\omega_d}{\omega_s} \quad (10)$$

$$\xi_{d,opt} = \frac{c_d}{2m_d\omega_d} \quad (11)$$

$\omega_{d,opt}$  is the frequency of the TMD as given in Eq. (12), and  $\omega_s$  is the frequency of single degree of freedom (SDOF) structure as shown in Eq. (13).

$$\omega_d = \sqrt{\frac{k_d}{m_d}} \quad (12)$$

$$\omega_s = \sqrt{\frac{k}{m}} \quad (13)$$

The equation of Den Hartog (1947) was found for harmonic excitations. Warburton (1982) derived the formulation for white-noise excitation. Sadek et al. (1997) used curve-fitting of numerical optimization results to find a formulation including the effect of inherent damping of the structure. Leung and Zhang (2009) employed PSO to find the TMD tuning equations.

## 2.3. ANN-based formulation for TMD optimization

Artificial neural networks (ANN) are an artificial intelligence modelling tool, and it based on simulating human brain behavior on different systems. Via ANN, it is possible estimate final results without processing the solving of problems. Yucel et al. (2019) developed an ANN model for

optimization of TMDs providing optimum period ( $T_d$ ) and damping ratio ( $\xi_{d,opt}$ ) of TMDs. The relationship between  $T_d$  and stiffness of TMD ( $k_d$ ) is given as Eq. (14).

$$T_d = 2\pi \sqrt{\frac{m_d}{k_d}} \quad (14)$$

The structure of the developed ANN model is shown as Fig. 3. The inputs are the period ( $T_s$ ) of SDOF structure given as Eq. (15) and mass ratio ( $\mu$ ).

$$T_s = 2\pi \sqrt{\frac{m}{k}} \quad (15)$$

In machine learning of the ANN model, FPA algorithm developed by Yang (2012) was used. The methodology given in Bekdaş et al. (2017) was used to find the optimum TMD values for different cases of SDOF structures. The developed ANN model was used to find the simplified equations given in Table 3.

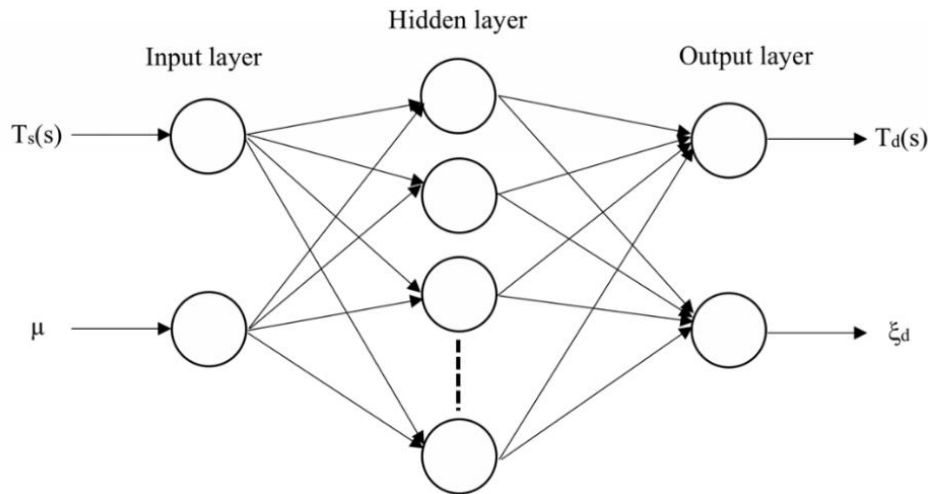


Fig. 3. ANN structure.

Table 3. ANN based formulations (Yucel et al., 2019).

| Method      | $f_{opt}$   | $\xi_{d,opt}$   |
|-------------|---|---|
| Linear      | $f_{opt} = -0.6438\mu + 0.9966$   | $\xi_{d,opt} = -0.5673\mu + 0.1235$   |
| Polynomial  | $f_{opt} = -249.91\mu^5 + 400.09\mu^4 - 208.03\mu^3 + 43.801\mu^2 - 4.1453\mu + 1.0675$ | $\xi_{d,opt} = -54.673\mu^4 + 54.639\mu^3 - 19.274\mu^2 + 3.2302\mu + 0.0237$ |
| Exponential | $f_{opt} = 1.0038 e^{-0.747\mu}$  | $\xi_{d,opt} = 1.1258 e^{2.8573\mu}$  |

### 3. Comparison of the methods

The investigations were done for four cases of SDOF structure period. All structures were also tested for three mass ratio values. The cases are shown in Tables 4 and 5.

The results of different methods are presented in Tables 6-9 including the sub-cases of mass ratio in Appendix. Also, the comparisons intended for  $x$ ,  $a$  and stroke values within these results of formulations, which were proposed in literature and developed as ANN-based, can be seen in Fig. 4-7 for case 1-4, respectively. The results include the optimum TMD values, maximum displacement ( $x$ ) and acceleration ( $a$ ) of the structure. Also, stroke values are presented for the TMD. The presented stroke value is a normalized value as given in Eq. (16).

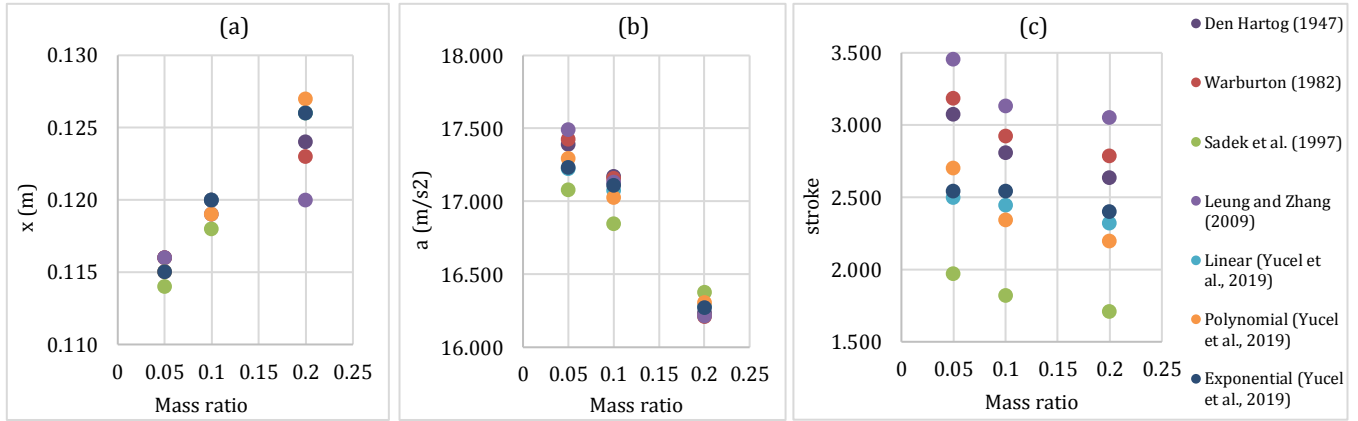
$$stroke = \frac{\max(|x_d|) - \max(|x|)}{\max(|x|)_{structure \text{ without TMD}}} \quad (16)$$

Table 4. Numerical cases for periods.

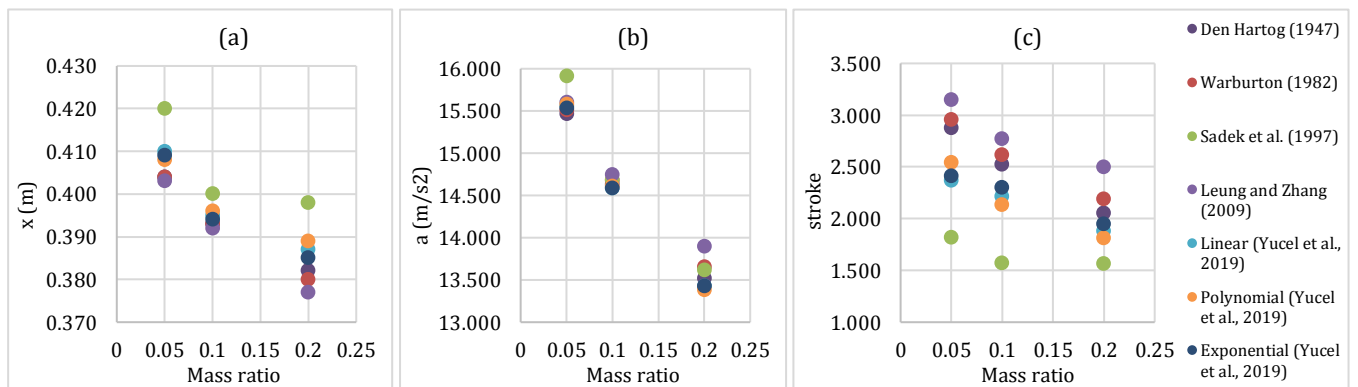
| Case | $T_s$ (s) |
|------|-----------|
| 1    | 0.5       |
| 2    | 1         |
| 3    | 2         |
| 4    | 4         |

Table 5. Numerical cases for mass ratio.

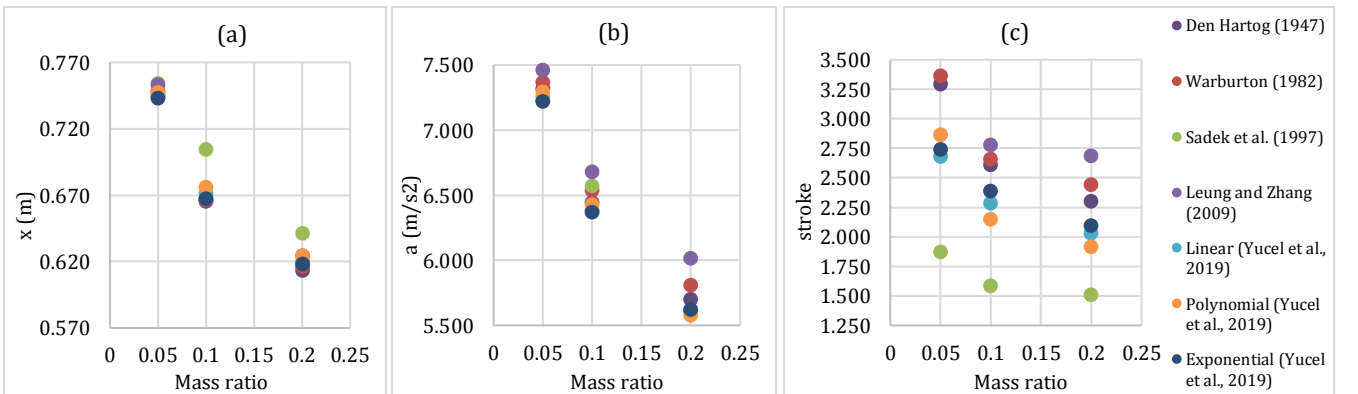
| Case | $\mu$ |
|------|-------|
| a    | 0.05  |
| b    | 0.10  |
| c    | 0.20  |



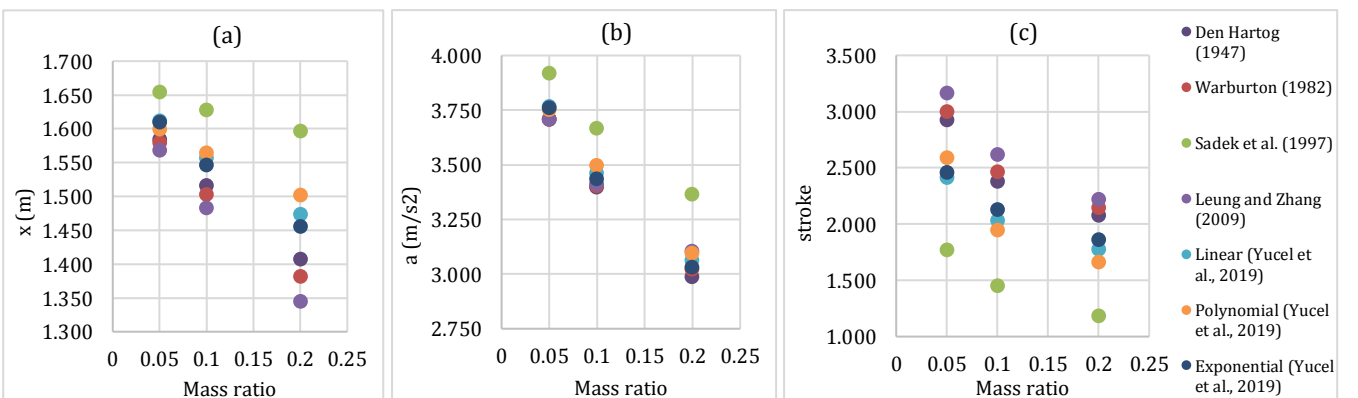
**Fig. 4.** The numerical results (Case 1): (a)  $x$  (m); (b)  $a$  (m/s<sup>2</sup>); (c) stroke.



**Fig. 5.** The numerical results (Case 2): (a)  $x$  (m); (b)  $a$  (m/s<sup>2</sup>); (c) stroke.



**Fig. 6.** The numerical results (Case 3): (a)  $x$  (m); (b)  $a$  (m/s<sup>2</sup>); (c) stroke.



**Fig. 7.** The numerical results (Case 4): (a)  $x$  (m); (b)  $a$  (m/s<sup>2</sup>); (c) stroke.

#### 4. Conclusions

In general, the ANN-based formulations are more effective than all formulations (except of Sadek et al. (1997)). When the optimum damping ratios of TMDs are investigated, Sadek et al. (1997) have a very big optimum value comparing to ANN-based method. Thus, the optimum TMD designed according to Sadek et al. (1997) have a high cost. By the increase of the damping, control can be provided with a small stroke value. The ANN-based methods have also small stroke values comparing to the others. By the increase of mass ratio, the damping values are increasing, and the values of Sadek et al. (1997) (4.5%) may need a high cost design. The effectiveness of the ANN-based formulations can be best seen

in average results of response under different excitations.

While ANN-based methods (Yucel et al., 2019) and Sadek et al. (1997) are the best in reduction of maximum responses, the classical methods like Den Hartog and Warburton may be more effective for long period structures. For the same cases of 4s period SDOF structure, the best method is PSO-based formulations (Leung and Zhang, 2009).

In total, the ANN-based method provides comparative solutions with a feasible TMD design. The success of optimum values is both seen for the displacement and the acceleration values. By the increase of the mass ratio, a small advantage of the exponential equations can be seen, while all three types have similar performances.

#### Appendix

**Table 6.** The numerical values (Case 1).

|             | SDOF<br>without<br>TMD  | Den<br>Hartog<br>(1947) | Warburton<br>(1982) | Sadek et al.<br>(1997) | Leung<br>and<br>Zhang<br>(2009) | Linear<br>(Yucel et al.,<br>2019) | Polynomial<br>(Yucel et al.,<br>2019) | Exponential<br>(Yucel et<br>al., 2019) |
|-------------|-------------------------|-------------------------|---------------------|------------------------|---------------------------------|-----------------------------------|---------------------------------------|--|
| $\mu=0.05$  |                         |                         |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 0.525                   | 0.532               | 0.531                  | 0.547                           | 0.518                             | 0.528                                 | 0.517                                  |
| $\xi_d$ (%) |                         | 10.911                  | 10.981              | 26.584                 | 10.914                          | 15.187                            | 14.351                                | 14.512                                 |
| Critical    | $x$ (m)                 | 0.124                   | 0.116               | 0.116                  | 0.114                           | 0.116                             | 0.115                                 | 0.115                                  |
|             | $a$ (m/s <sup>2</sup> ) | 19.761                  | 17.391              | 17.426                 | 17.077                          | 17.492                            | 17.295                                | 17.233                                 |
|             | stroke                  |                         | 3.076               | 3.185                  | 1.972                           | 3.457                             | 2.499                                 | 2.544                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |
| Mean        | $x$ (m)                 | 0.051                   | 0.048               | 0.048                  | 0.049                           | 0.048                             | 0.048                                 | 0.048                                  |
|             | $a$ (m/s <sup>2</sup> ) | 8.119                   | 7.280               | 7.284                  | 7.314                           | 7.308                             | 7.268                                 | 7.267                                  |
|             | stroke                  |                         | 2.380               | 2.428                  | 1.540                           | 2.543                             | 1.999                                 | 2.033                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |
| $\mu=0.10$  |                         |                         |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 0.550                   | 0.564               | 0.558                  | 0.587                           | 0.536                             | 0.543                                 | 0.537                                  |
| $\xi_d$ (%) |                         | 15.076                  | 15.273              | 34.697                 | 15.140                          | 18.023                            | 20.315                                | 16.741                                 |
| Critical    | $x$ (m)                 | 0.124                   | 0.120               | 0.120                  | 0.118                           | 0.119                             | 0.120                                 | 0.120                                  |
|             | $a$ (m/s <sup>2</sup> ) | 19.761                  | 17.169              | 17.158                 | 16.844                          | 17.133                            | 17.073                                | 17.109                                 |
|             | stroke                  |                         | 2.806               | 2.924                  | 1.821                           | 3.130                             | 2.446                                 | 2.542                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |
| Mean        | $x$ (m)                 | 0.051                   | 0.048               | 0.048                  | 0.049                           | 0.047                             | 0.049                                 | 0.049                                  |
|             | $a$ (m/s <sup>2</sup> ) | 8.119                   | 6.891               | 6.896                  | 7.010                           | 6.911                             | 6.886                                 | 6.885                                  |
|             | stroke                  |                         | 2.025               | 2.094                  | 1.357                           | 2.216                             | 1.800                                 | 1.863                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |
| $\mu=0.20$  |                         |                         |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 0.600                   | 0.632               | 0.613                  | 0.695                           | 0.576                             | 0.564                                 | 0.578                                  |
| $\xi_d$ (%) |                         | 20.412                  | 20.972              | 44.991                 | 20.707                          | 23.696                            | 24.842                                | 22.277                                 |
| Critical    | $x$ (m)                 | 0.124                   | 0.124               | 0.123                  | 0.126                           | 0.120                             | 0.126                                 | 0.126                                  |
|             | $a$ (m/s <sup>2</sup> ) | 19.761                  | 16.234              | 16.211                 | 16.378                          | 16.212                            | 16.284                                | 16.273                                 |
|             | stroke                  |                         | 2.636               | 2.787                  | 1.710                           | 3.054                             | 2.321                                 | 2.402                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |
| Mean        | $x$ (m)                 | 0.051                   | 0.049               | 0.047                  | 0.050                           | 0.046                             | 0.050                                 | 0.049                                  |
|             | $a$ (m/s <sup>2</sup> ) | 8.119                   | 6.326               | 6.317                  | 6.585                           | 6.382                             | 6.369                                 | 6.360                                  |
|             | stroke                  |                         | 1.841               | 1.945                  | 1.253                           | 2.170                             | 1.627                                 | 1.682                                  |
|             |                         |                         |                     |                        |                                 |                                   |                                       |  |

**Table 7.** The numerical values (Case 2).

|            |                         | SDOF<br>without<br>TMD | Den Har-<br>tog<br>(1947) | Warburton<br>(1982) | Sadek et al.<br>(1997) | Leung<br>and<br>Zhang<br>(2009) | Linear<br>(Yucel et al.,<br>2019) | Polynomial<br>(Yucel et al.,<br>2019) | Exponential<br>(Yucel et<br>al., 2019) |
|------------|-------------------------|------------------------|---------------------------|---------------------|------------------------|---------------------------------|-----------------------------------|---------------------------------------|--|
| $\mu=0.05$ |                         |                        |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               |                        | 1.050                     | 1.063               | 1.062                  | 1.093                           | 1.037                             | 1.057                                 | 1.034                                  |
|            | $\xi_d$ (%)             |                        | 10.911                    | 10.981              | 26.584                 | 10.914                          | 15.187                            | 14.351                                | 14.512                                 |
| Critical   | $x$ (m)                 | 0.454                  | 0.404                     | 0.404               | 0.420                  | 0.403                           | 0.410                             | 0.408                                 | 0.409                                  |
|            | $a$ (m/s <sup>2</sup> ) | 17.987                 | 15.466                    | 15.510              | 15.915                 | 15.604                          | 15.560                            | 15.582                                | 15.532                                 |
|            | stroke                  |                        | 2.878                     | 2.959               | 1.821                  | 3.149                           | 2.368                             | 2.543                                 | 2.414                                  |
| Mean       | $x$ (m)                 | 0.165                  | 0.142                     | 0.142               | 0.150                  | 0.141                           | 0.145                             | 0.144                                 | 0.145                                  |
|            | $a$ (m/s <sup>2</sup> ) | 6.553                  | 5.448                     | 5.457               | 5.684                  | 5.490                           | 5.512                             | 5.506                                 | 5.500                                  |
|            | stroke                  |                        | 2.297                     | 2.326               | 1.468                  | 2.402                           | 1.940                             | 2.044                                 | 1.976                                  |
| $\mu=0.10$ |                         |                        |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               |                        | 1.100                     | 1.129               | 1.117                  | 1.174                           | 1.073                             | 1.086                                 | 1.073                                  |
|            | $\xi_d$ (%)             |                        | 15.076                    | 15.273              | 34.697                 | 15.140                          | 18.023                            | 20.315                                | 16.741                                 |
| Critical   | $x$ (m)                 | 0.454                  | 0.393                     | 0.393               | 0.400                  | 0.392                           | 0.395                             | 0.396                                 | 0.394                                  |
|            | $a$ (m/s <sup>2</sup> ) | 17.987                 | 14.630                    | 14.675              | 14.687                 | 14.744                          | 14.590                            | 14.614                                | 14.589                                 |
|            | stroke                  |                        | 2.524                     | 2.618               | 1.573                  | 2.770                           | 2.217                             | 2.132                                 | 2.299                                  |
| Mean       | $x$ (m)                 | 0.165                  | 0.139                     | 0.138               | 0.144                  | 0.137                           | 0.141                             | 0.141                                 | 0.140                                  |
|            | $a$ (m/s <sup>2</sup> ) | 6.553                  | 5.138                     | 5.142               | 5.283                  | 5.182                           | 5.146                             | 5.154                                 | 5.142                                  |
|            | stroke                  |                        | 1.814                     | 1.855               | 1.201                  | 1.932                           | 1.622                             | 1.555                                 | 1.680                                  |
| $\mu=0.20$ |                         |                        |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               |                        | 1.200                     | 1.265               | 1.225                  | 1.389                           | 1.152                             | 1.128                                 | 1.157                                  |
|            | $\xi_d$ (%)             |                        | 20.412                    | 20.972              | 44.991                 | 20.707                          | 23.696                            | 24.842                                | 22.277                                 |
| Critical   | $x$ (m)                 | 0.454                  | 0.382                     | 0.380               | 0.398                  | 0.377                           | 0.387                             | 0.389                                 | 0.385                                  |
|            | $a$ (m/s <sup>2</sup> ) | 17.987                 | 13.520                    | 13.654              | 13.620                 | 13.899                          | 13.425                            | 13.384                                | 13.430                                 |
|            | stroke                  |                        | 2.054                     | 2.188               | 1.567                  | 2.502                           | 1.882                             | 1.812                                 | 1.948                                  |
| Mean       | $x$ (m)                 | 0.165                  | 0.136                     | 0.134               | 0.142                  | 0.133                           | 0.138                             | 0.139                                 | 0.138                                  |
|            | $a$ (m/s <sup>2</sup> ) | 6.553                  | 4.756                     | 4.786               | 4.849                  | 4.882                           | 4.731                             | 4.731                                 | 4.731                                  |
|            | stroke                  |                        | 1.519                     | 1.587               | 1.041                  | 1.746                           | 1.354                             | 1.290                                 | 1.400                                  |

**Table 8.** The numerical values (Case 3).

|            | SDOF with-<br>out TMD   | Den Har-<br>tog<br>(1947) | Warburton<br>(1982) | Sadek et al.<br>(1997) | Leung<br>and<br>Zhang<br>(2009) | Linear<br>(Yucel et al.,<br>2019) | Polynomial<br>(Yucel et al.,<br>2019) | Exponential<br>(Yucel et<br>al., 2019) |
|------------|-------------------------|---------------------------|---------------------|------------------------|---------------------------------|-----------------------------------|---------------------------------------|--|
| $\mu=0.05$ |                         |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               | 2.100                     | 2.127               | 2.123                  | 2.186                           | 2.074                             | 2.114                                 | 2.068                                  |
|            | $\xi_d$ (%)             | 10.911                    | 10.981              | 26.584                 | 10.914                          | 15.187                            | 14.351                                | 14.512                                 |
| Critical   | $x$ (m)                 | 0.830                     | 0.745               | 0.748                  | 0.754                           | 0.753                             | 0.744                                 | 0.743                                  |
|            | $a$ (m/s <sup>2</sup> ) | 8.237                     | 7.316               | 7.364                  | 7.258                           | 7.458                             | 7.223                                 | 7.218                                  |
|            | stroke                  |                           | 3.291               | 3.362                  | 1.872                           | 3.524                             | 2.678                                 | 2.737                                  |
| Mean       | $x$ (m)                 | 0.382                     | 0.344               | 0.344                  | 0.356                           | 0.343                             | 0.348                                 | 0.348                                  |
|            | $a$ (m/s <sup>2</sup> ) | 3.788                     | 3.303               | 3.311                  | 3.392                           | 3.329                             | 3.325                                 | 3.320                                  |
|            | stroke                  |                           | 2.337               | 2.374                  | 1.485                           | 2.465                             | 1.975                                 | 2.011                                  |
| $\mu=0.10$ |                         |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               | 2.200                     | 2.257               | 2.234                  | 2.349                           | 2.145                             | 2.173                                 | 2.147                                  |
|            | $\xi_d$ (%)             | 15.076                    | 15.273              | 34.697                 | 15.140                          | 18.023                            | 20.315                                | 16.741                                 |
| Critical   | $x$ (m)                 | 0.830                     | 0.665               | 0.666                  | 0.704                           | 0.667                             | 0.671                                 | 0.667                                  |
|            | $a$ (m/s <sup>2</sup> ) | 8.237                     | 6.442               | 6.533                  | 6.571                           | 6.677                             | 6.376                                 | 6.369                                  |
|            | stroke                  |                           | 2.608               | 2.655                  | 1.586                           | 2.776                             | 2.281                                 | 2.385                                  |
| Mean       | $x$ (m)                 | 0.382                     | 0.328               | 0.327                  | 0.346                           | 0.325                             | 0.333                                 | 0.331                                  |
|            | $a$ (m/s <sup>2</sup> ) | 3.788                     | 3.053               | 3.072                  | 3.178                           | 3.108                             | 3.065                                 | 3.055                                  |
|            | stroke                  |                           | 1.902               | 1.950                  | 1.253                           | 2.045                             | 1.692                                 | 1.755                                  |
| $\mu=0.20$ |                         |                           |                     |                        |                                 |                                   |                                       |  |
|            | $T_d$ (s)               | 2.400                     | 2.530               | 2.450                  | 2.779                           | 2.305                             | 2.256                                 | 2.313                                  |
|            | $\xi_d$ (%)             | 20.412                    | 20.972              | 44.991                 | 20.707                          | 23.696                            | 24.842                                | 22.277                                 |
| Critical   | $x$ (m)                 | 0.830                     | 0.613               | 0.616                  | 0.641                           | 0.624                             | 0.624                                 | 0.618                                  |
|            | $a$ (m/s <sup>2</sup> ) | 8.237                     | 5.700               | 5.808                  | 5.611                           | 6.016                             | 5.574                                 | 5.621                                  |
|            | stroke                  |                           | 2.301               | 2.442                  | 1.508                           | 2.685                             | 2.026                                 | 2.094                                  |
| Mean       | $x$ (m)                 | 0.382                     | 0.310               | 0.307                  | 0.335                           | 0.305                             | 0.318                                 | 0.316                                  |
|            | $a$ (m/s <sup>2</sup> ) | 3.788                     | 2.751               | 2.798                  | 2.877                           | 2.893                             | 2.741                                 | 2.735                                  |
|            | stroke                  |                           | 1.600               | 1.670                  | 1.064                           | 1.811                             | 1.424                                 | 1.472                                  |



**Table 9.** The numerical values (Case 4).

|             | SDOF with-<br>out TMD   | Den Har-<br>tog<br>(1947) | Warburton<br>(1982) | Sadek et al.<br>(1997) | Leung<br>and<br>Zhang<br>(2009) | Linear<br>(Yucel et al.,<br>2019) | Polynomial<br>(Yucel et al.,<br>2019) | Exponential<br>(Yucel et<br>al., 2019) |
|-------------|-------------------------|---------------------------|---------------------|------------------------|---------------------------------|-----------------------------------|---------------------------------------|--|
| $\mu=0.05$  |                         |                           |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 4.200                     | 4.254               | 4.246                  | 4.372                           | 4.148                             | 4.228                                 | 4.137                                  |
| $\xi_d$ (%) |                         | 10.911                    | 10.981              | 26.584                 | 10.914                          | 15.187                            | 14.351                                | 14.512                                 |
| Critical    | $x$ (m)                 | 1.936                     | 1.585               | 1.580                  | 1.655                           | 1.569                             | 1.612                                 | 1.610                                  |
|             | $a$ (m/s <sup>2</sup> ) | 4.797                     | 3.707               | 3.707                  | 3.918                           | 3.707                             | 3.753                                 | 3.760                                  |
|             | stroke                  |                           | 2.928               | 2.999                  | 1.770                           | 3.165                             | 2.589                                 | 2.461                                  |
| Mean        | $x$ (m)                 | 0.639                     | 0.591               | 0.591                  | 0.599                           | 0.590                             | 0.593                                 | 0.592                                  |
|             | $a$ (m/s <sup>2</sup> ) | 1.591                     | 1.448               | 1.450                  | 1.450                           | 1.454                             | 1.444                                 | 1.443                                  |
|             | stroke                  |                           | 2.209               | 2.226                  | 1.364                           | 2.271                             | 1.861                                 | 1.901                                  |
| $\mu=0.10$  |                         |                           |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 4.400                     | 4.514               | 4.467                  | 4.698                           | 4.291                             | 4.346                                 | 4.294                                  |
| $\xi_d$ (%) |                         | 15.076                    | 15.273              | 34.697                 | 15.140                          | 18.023                            | 20.315                                | 16.741                                 |
| Critical    | $x$ (m)                 | 1.936                     | 1.517               | 1.503                  | 1.628                           | 1.483                             | 1.557                                 | 1.547                                  |
|             | $a$ (m/s <sup>2</sup> ) | 4.797                     | 3.399               | 3.403                  | 3.667                           | 3.412                             | 3.462                                 | 3.437                                  |
|             | stroke                  |                           | 2.378               | 2.466                  | 1.451                           | 2.619                             | 1.950                                 | 2.130                                  |
| Mean        | $x$ (m)                 | 0.639                     | 0.571               | 0.570                  | 0.582                           | 0.570                             | 0.573                                 | 0.573                                  |
|             | $a$ (m/s <sup>2</sup> ) | 1.591                     | 1.372               | 1.378                  | 1.367                           | 1.388                             | 1.364                                 | 1.365                                  |
|             | stroke                  |                           | 1.710               | 1.732                  | 1.080                           | 1.781                             | 1.535                                 | 1.593                                  |
| $\mu=0.20$  |                         |                           |                     |                        |                                 |                                   |                                       |  |
| $T_d$ (s)   |                         | 4.800                     | 5.060               | 4.900                  | 5.558                           | 4.609                             | 4.513                                 | 4.627                                  |
| $\xi_d$ (%) |                         | 20.412                    | 20.972              | 44.991                 | 20.707                          | 23.696                            | 24.842                                | 22.277                                 |
| Critical    | $x$ (m)                 | 1.936                     | 1.408               | 1.382                  | 1.597                           | 1.345                             | 1.474                                 | 1.456                                  |
|             | $a$ (m/s <sup>2</sup> ) | 4.797                     | 2.989               | 3.023                  | 3.366                           | 3.104                             | 3.065                                 | 3.033                                  |
|             | stroke                  |                           | 2.077               | 2.148                  | 1.185                           | 2.218                             | 1.775                                 | 1.863                                  |
| Mean        | $x$ (m)                 | 0.639                     | 0.542               | 0.540                  | 0.563                           | 0.537                             | 0.548                                 | 0.546                                  |
|             | $a$ (m/s <sup>2</sup> ) | 1.591                     | 1.264               | 1.276                  | 1.252                           | 1.302                             | 1.251                                 | 1.253                                  |
|             | stroke                  |                           | 1.329               | 1.358                  | 0.859                           | 1.423                             | 1.194                                 | 1.236                                  |

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