



Research Article

Modification of the effective area method on two-way loaded shallow foundations

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ABSTRACT

In rectangular/square based and two-way loaded (two-way eccentric) shallow foundations, four zones in which the resultant load might act are defined in the effective area method. Three out of the four zones that are employed in the determination of the effective areas overlap around kern. Only one zone that has a triangular-shaped effective area (called as case 1 in the literature) out of the four zones has no overlap with the others. The resultant load will always be out of the kern for case 1, and also it might be out of the kern for the remaining three cases. Design of foundations is not acceptable in general if the resultant load acts out of the kern. In the present study, the four cases are reconsidered. The zones on which the resultant load can be acting for the four cases are modified because these zones are overlapped partly. The modification has been made to have clear borders between the zones. On top of that, zone 4 is divided into two. A new zone corresponding to the area of kern is defined as zone 5. The design will be accepted if the resultant load acts within zone 5 (the kern). Also, the graphs in use to determine the dimensions of the effective areas are eliminated since it is not precise. Formulas are derived to determine the dimensions of the effective areas instead of using the graphs. Two new criteria are discovered and proposed to check whether the resultant load acts outside, inside or on the borderline of zone 5 (the kern).

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1. Introduction

The steps on the geotechnical design of two-way loaded (two-way eccentric) foundations are as follows:

a. Determine the eccentricities of e_B and e_L seen in Fig. 1 in both directions of B and L , respectively.

b. Determine the dimensions of the kern as seen in Fig. 2, and check the location of the resultant load (Q) whether it acts inside, outside or on the borderline of the kern.

Use the criteria seen below to find out the location of the resultant load (Eq.(1)).

$$\left(\frac{6e_B}{B} + \frac{6e_L}{L}\right) \leq 1 \quad (1)$$

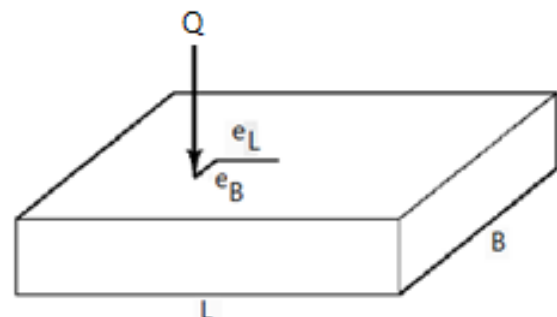


Fig. 1. Two-way loaded square/rectangular foundation.

If Eq. (1), which represents the last two terms the second parenthesis of Eq. (2) has the value equal to one, the minimum bearing pressure would be nil whereas if the

value is more than one, it would be a negative number, which means the minimum bearing is tension. In other words, if Eq. (1) is satisfied, the minimum bearing pressure is either compression or nil. Otherwise, the minimum bearing is a negative value that reflects a gap between the base of the foundation and underlying soil. Since no gap in the design is accepted under any foundation, there must be some solutions to avoid this situation like increasing dimensions of foundation or reducing the eccentricities physically.

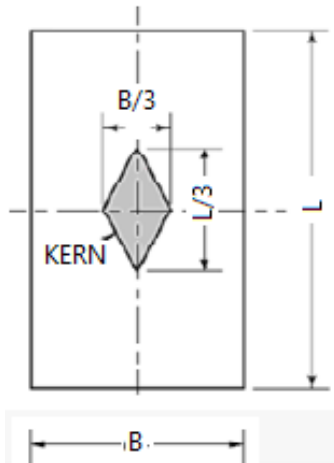


Fig. 2. Dimensions of the kern.

c. Use the sign (+ or -) in Eq. (2) to calculate the maximum and minimum bearing pressures at the base of foundation according to the position of the load (inside, or on the borderline of the kern).

$$q = \left(\frac{Q+W_f}{BL} - u_D \right) \left(1 \pm \frac{6e_B}{B} \pm \frac{6e_L}{L} \right) \quad (2)$$

where

- Q is load from superstructure,
- W_f is weight of foundation,
- B and L are width and length of foundation,
- u_D is pore water pressure at the base level of the foundation,
- e_B and e_L are eccentricities on B and L directions, respectively.

In the design, the maximum bearing pressure must be less than or equal to the allowable bearing capacity of soil, and at the same time, the minimum bearing pressure must be compression or at least zero but not tension. Once, the maximum and minimum bearing pressures are estimated, allowable bearing capacity of the foundation must be determined too.

The bearing capacity of foundation can be estimated by several methods. Terzaghi’s bearing capacity equation (Terzaghi, 1943) is widely used to estimate the ultimate bearing capacity of foundations even it has many limitations in practical applications. To minimize these limitations, many proposals have been made by researchers (Meyerhof, 1963; Prakash and Saran, 1971; Saran and Agarwal, 1993; Vesic, 1973; Sawwaf and Nazir, 2012;

Krabbenhof et al., 2012; Loukidis et al., 2008, Badakhshan and Noorzad, 2017, Sahu et al., 2019). One of these methods has been proposed by Meyerhof (1963) and his proposed method is known as “Meyerhof Bearing Capacity Equation, General Bearing Capacity Equation, or The Effective Area Method”. Vesic (1973) has also proposed an equation almost identical with Meyerhof’s equation. Meyerhof’s general bearing capacity equation is seen below:

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + 0.5 \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \quad (3)$$

where

- N_c, N_q, N_γ are bearing capacity factors,
- $F_{cs}, F_{qs}, F_{\gamma s}$ are the shape factors,
- $F_{cd}, F_{qd}, F_{\gamma d}$ are the depth factors,
- $F_{ci}, F_{qi}, F_{\gamma i}$ are the inclination factors,
- c' is cohesion,
- q is effective stress at the base level of foundation,
- B' is the effective width of foundation,
- γ is unit weight of soil.

Then, the ultimate bearing load can be estimated as follows.

$$Q_{ult} = q_u A' \quad (4)$$

A procedure to determine the effective area (A') and effective width (B') that would be used in the Meyerhof’s general bearing capacity equation was proposed by (Highter and Anders, 1985) in addition to one proposed by Meyerhof (1963). It is the common practice to employ the proposed “effective area” determination on the design of two-way loaded foundations. In the determination of effective area, four cases are provided to design of square/rectangular based, and two-way loaded foundations by Highter and Anders in 1985. In the determination of the four cases, the criteria are the ranges of the ratios of e_B/B , and e_L/L . In general, nothing is mentioned for these four cases about the application points of the resultant loads whether it is in or out of the kern. However, anything may occur in terms of eccentricity. It means that the resultant force can be in, out or on the borderline of the kern except case 1 where eccentricity is always out of the kern.

2. Cases Defined in the Effective Area Method

When one has a closer look into the four cases mentioned in the event of effective area method, the resultant load is always out of the kern in case 1 seen in Fig. 3, mostly out of the kern in cases 2 (Fig. 4), and 3 (Fig. 5), some area out of the kern even in case 4 (Fig. 6). Thus, the cases should be modified to have a clear border between the cases. Only case 1 has the areas not overlap with the areas of the rest of the cases. There are overlaps of the areas in the cases of 2, 3, and 4 as seen in Figs. 4, 5, and 6. Zone 4 seen in Fig. 6 is the common zone in the cases of 2, 3, and 4. The shapes and borders of the effective areas are taken from Das (2007).

CASE 1: $[(e_L/L) > (1/6) \text{ and } (e_B/B) > (1/6)]$

As it is seen in Fig. 3, the resultant acts within the zones 1. It is obvious that the resultant load acts out of the kern so that minimum bearing pressure would be tension and a gap between the base of the foundation and the underlying soil would occur.

CASE 2: $[(e_L/L) < 0.5 \text{ and } 0 < (e_B/B) < (1/6)]$

As it is seen in Fig. 4, the resultant load acts within zone 2. Again, it is obvious that the resultant load is not always acting within or border line but mostly out of the kern so that minimum bearing pressure would be tension mostly and a gap between the base of foundation and the underlying soil would occur.

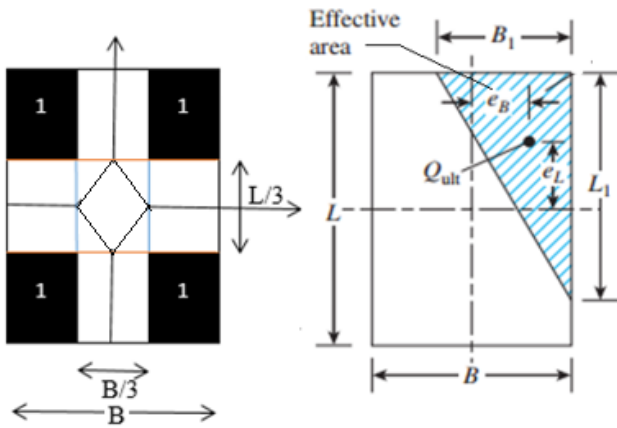


Fig. 3. Resultant load acts in zone 1 (Case 1).

CASE 3: $[(e_L/L) < (1/6) \text{ and } 0 < (e_B/B) < 0.5]$

As it is seen in Fig. 5, the resultant load acts within zone 3. Again, the resultant load not always acts within or on the borderline but mostly out of the kern. Hence, minimum bearing pressure would be tension mostly and a gap between the base of the foundation and the underlying soil occurred.

CASE 4: $[(e_L/L) < (1/6) \text{ and } (e_B/B) < (1/6)]$

As it is seen in Fig. 6, the resultant load acts within zone 4. It acts within the kern or on its borderline, but also out of it. Thus, the minimum bearing pressure might be tension and a gap between the base of the foundation and the underlying soil might occur.

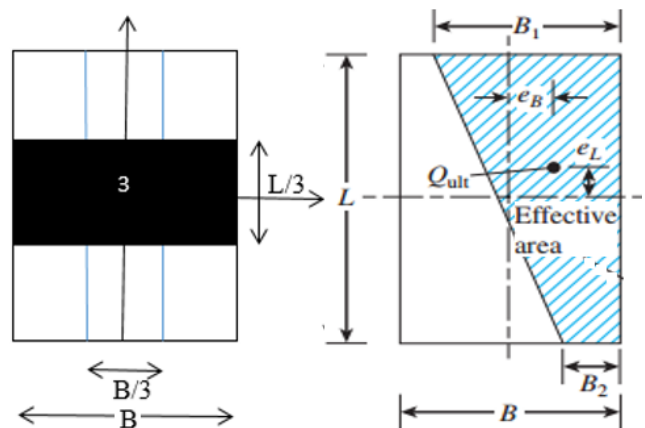


Fig. 5. Resultant load acts in zone 3 (Case 3).

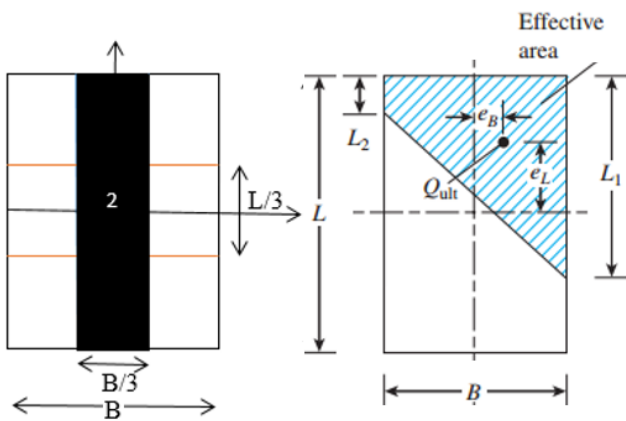


Fig. 4. Resultant load acts in zone 2 (Case 2).

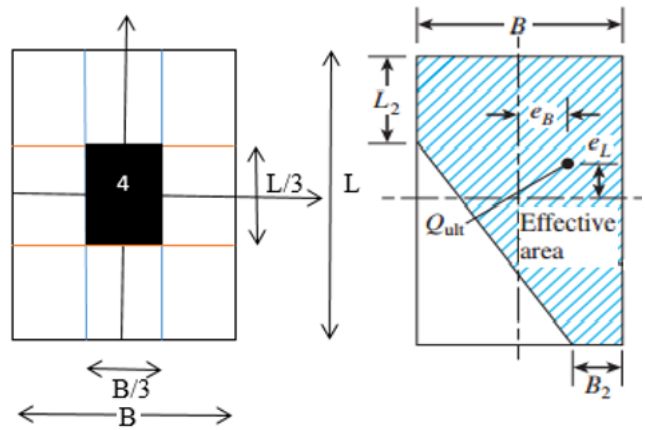


Fig. 6. Resultant load acts in zone 4 (Case 4).

3. Modified Cases

CASE 1 has not changed so that the resultant load acts out of the kern all the time, while it is more often than out of the kern in cases of 2, 3, and 4. In the modified cases of 2, 3, and 4, the resultant load is out of the kern also. Thus, these cases may be studied in any research or other purposes except the design of foundations that would be applied in the field. The reason for that there would be tension between the base of foundation and underlying soil.

In this study, a new case is defined as “case 5” (Fig. 11) that is actually representing the kern so that case 5 can

be used to design foundations that would be applied in the field because the resultant load acts in the kern so that the minimum bearing pressure would be a positive value or at least nil.

Modified CASE 2: Modified and redefined as seen below.
 $[(1/6) < (e_L/L) < 0.5 \text{ and } 0 < (e_B/B) < (1/6)]$

When these ranges are applied to zone 2, it will become as seen in Fig. 7. The comparison between Figs. 4 and 7 shows the difference.

The dimensions of the effective area seen in Fig. 4b can be calculated by the Eqs. (5 to 9) instead of using the graph generated in the original effective area method.

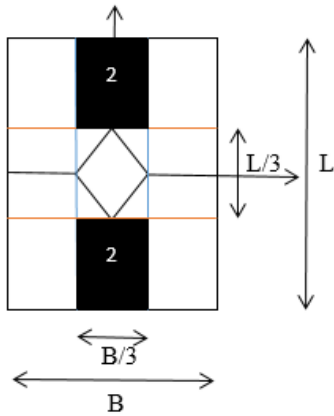


Fig. 7. Zone 2 after the modification.

$$A_o = \frac{B-6e_B}{B+6e_B} \tag{5}$$

$$L_1 = \left(\frac{1.5-3e_L/L}{1+A_o+A_o^2} \right) (A_o + 1)L \leq L \tag{6}$$

$$L_2 = A_o L_1 \leq L \tag{7}$$

The effective area;

$$A' = \frac{1}{2}(L_1 + L_2)B \tag{8}$$

and the effective width;

$$B' = \frac{A'}{L_1 \text{ or } L_2 (\text{larger one})} \tag{9}$$

Modified CASE 3: Modified and redefined as seen below.

$$[(e_L/L) < (1/6) \text{ and } (1/6) < (e_B/B) < 0.5]$$

When these ranges are applied to zone 3, it will become as seen in Fig.8. The difference can be seen by comparing Figs. 5 and 8.

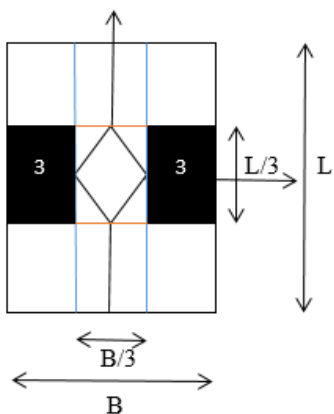


Fig. 8. Zone 3 after the modification.

The dimensions of the effective area seen in Fig. 4b can be calculated by the Eqs. (10 to 14) instead of using the graph generated in the original effective area method.

$$A_1 = \frac{L-6e_L}{L+6e_L} \tag{10}$$

$$B_1 = \left(\frac{1.5-3e_B/B}{1+A_1+A_1^2} \right) (A_1 + 1)B \leq B \tag{11}$$

$$B_2 = A_1 B_1 \leq B \tag{12}$$

The effective area;

$$A' = \frac{1}{2}(B_1 + B_2)L \tag{13}$$

and the effective width;

$$B' = \frac{A'}{L} \tag{14}$$

Modified CASE 4: The four dark triangular zones seen in Fig. 9 represent zone 4. The eccentricity of the resultant load would be within the following borders for this case.

$$e > e_{\max}, \text{ and } \frac{e_B}{B} \leq \frac{1}{6} \text{ and } \frac{e_L}{L} \leq \frac{1}{6}$$

where e and e_{\max} are calculated from Eqs. (18 and 19), respectively. The effective area and effective width for this case would be calculated just like the procedure given in case 5.

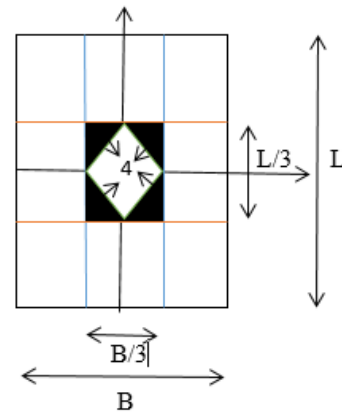


Fig. 9. Zone 4 after the modification.

CASE 5 (New): A new case that envisages the zone of kern exclusively considered (see Fig. 10). Two criteria for this zone have been determined and given below. One of the criteria can be employed to determine whether the resultant load is within, on or out of the borderline of kern.

Criteria 1:

The kern is seen in Fig. 11a (zone 5), and one of the four parts of the kern is shown in Fig. 11b. To create a criterion that would be employed for the case 5 in the effective area method, the steps are as follows.

1. Find angles α , β , and γ in Fig. 11b as seen below.

$$\alpha = \tan^{-1} \left(\frac{B}{L} \right) \tag{15}$$

$$\beta = \tan^{-1} \left(\frac{e_B}{e_L} \right) \tag{16}$$

$$\gamma = 180 - \alpha - \beta \tag{17}$$

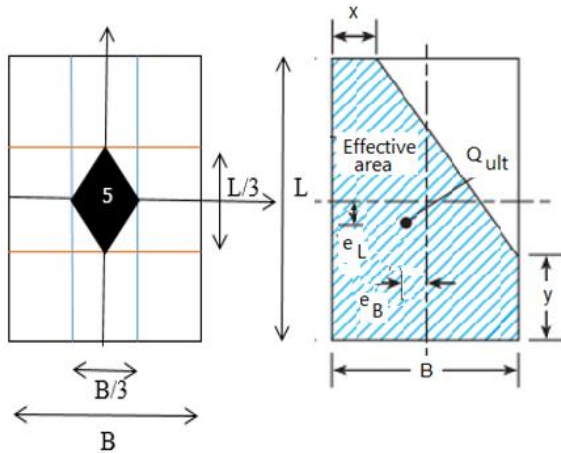


Fig. 10. Zone 5 newly defined.

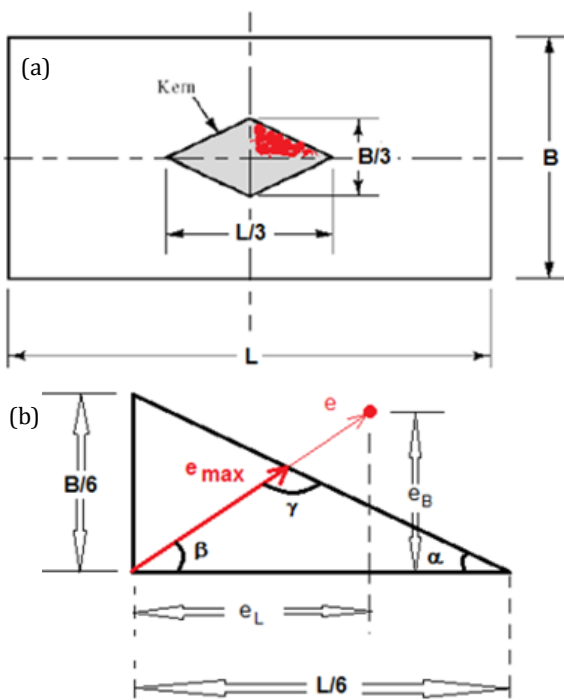


Fig. 11. The value of eccentricities on a foundation.

2. Find maximum value of eccentricity within the kern:

$$e_{max} = \frac{\sin \alpha}{\sin \gamma} \cdot \frac{L}{6} \tag{18}$$

3. Find the existing eccentricity;

$$e = \sqrt{(e_L^2 + e_B^2)} \tag{19}$$

Application of resultant load is within the kern if

$$e < e_{max} .$$

Application of resultant load is on the borderline of the kern if

$$e = e_{max} .$$

Application of resultant load is outside of the kern if

$$e > e_{max} .$$

Finally, check up on criteria of the modified cases should e be of less value than e_{max} or equal to it, the technique met in case 5 applies.

Criteria 2:

To have the point of application of the resultant load within the kern, with the eccentricity on B -direction only, the maximum value of $e_B = B/6$ while the eccentricity on L -direction is equal to zero ($e_L = 0$); or vice versa with the eccentricity on L -direction only (see Fig. 12).

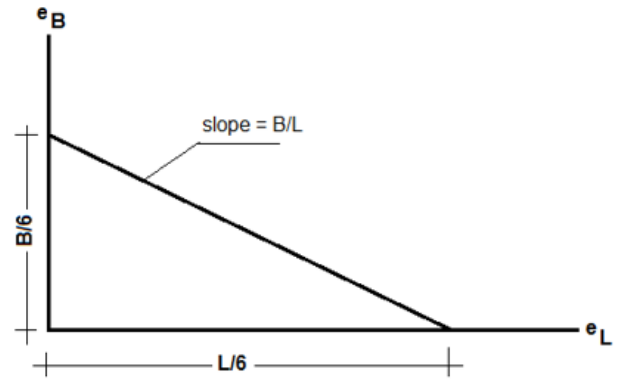


Fig. 12. One fourth of the kern.

Thus, the following relationships can be written:

$$\left[0 \leq \frac{e_B}{B} \leq \left(\frac{1}{6} - \frac{e_L}{L} \right) \right] \tag{20}$$

or

$$\left[0 \leq \frac{e_L}{L} \leq \left(\frac{1}{6} - \frac{e_B}{B} \right) \right] \tag{21}$$

If this is the case of eccentricities, the resultant load acts either in the kern or on its borderline. When this criterion is not satisfied, the resultant load is out of the kern, and the minimum value of the bearing pressure is a tension. In general, a foundation with tension under its base is not allowed to be constructed since there would be a gap between its base and underlying soil.

If the resultant load acts in zone 5, which is actually the kern, the shape of the effective area would be similar to one seen in Fig. 13b. Coordinates of the effective area can be located from 0 to 5 that must be numbered clockwise as seen in Fig 13a. The coordinates of the points (0 to 5) are numbered as follows:

$$0(x_0, y_0), 1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4), 5(x_5, y_5)$$

Actually $x_0 = x_5$, and $y_0 = y_5$. From Fig. 13b:

$$0(0,0), 1(0, L), 2(x, L), 3(B, y), 4(B,0), 5(0,0) \text{ can be written.}$$

Coordinates (C_x , and C_y) of the centroid of effective area can be written as follows (see Fig. 13b):

$$C_x = \frac{B}{2} - e_B \tag{22}$$

$$C_y = \frac{L}{2} - e_L \tag{23}$$

Also, these coordinates can be written as follows:

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) \tag{24}$$

$$C_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \quad (25)$$

$$C_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \quad (26)$$

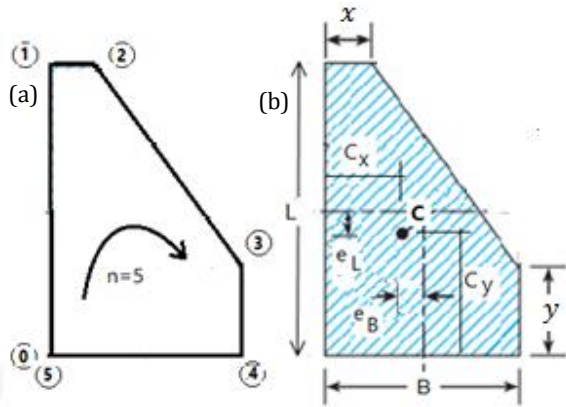


Fig. 13. Coordinates of effective area.

The right side of Eq. (25) is equal to the right side of Eq. (22), and Eq. (26) is equal to Eq. (23). Therefore, Eqs. (27) and (28) can be derived as follows:

$$-x^2L + (x + B)(xy - BL) - 2B^2y - 3\left(\frac{B}{2} - e_B\right)(-xL + xy - BL - By) = 0 \quad (27)$$

$$2xL^2 + (L + y)(xy - BL) - By^2 - 3\left(\frac{L}{2} - e_L\right)(-xL + xy - BL - By) = 0 \quad (28)$$

Eqs. (27) and (28) can be solved numerically for unknown x and y by a proper method. In this study, a MATLAB code has been developed to solve these equations and normalized values are given in Table 1. The effective area can be determined from:

$$A' = \frac{1}{2}(BL + xL + By - xy) \quad (29)$$

or

$$A' = \frac{1}{2}(B + x)(L - y) + By \quad (30)$$

or

$$A' = BL - \frac{1}{2}(B - x)(L - y) \quad (31)$$

and the effective width;

$$vB' = \frac{A'}{L} \quad (32)$$

Tables 1 and 2 can be used to determine effective areas for case 4 just like newly described case 5.

Table 1. x/B values.

e_L/L	0.027	0.053	0.080	0.107	0.133	0.160
e_B/B	x/B					
0.02	0.4614	--	--	--	--	--
0.04	0.7380	0.3408	0.0625	--	--	--
0.06	0.7323	0.5006	0.2389	0.0519	--	--
0.08	0.6976	0.5352	0.3303	0.1491	0.0124	--
0.10	0.6539	0.5250	0.3628	0.2027	0.0688	--
0.12	0.6090	0.4982	0.3633	0.2244	0.1000	--
0.14	0.5636	0.4642	0.3475	0.2259	0.1125	0.0144
0.16	0.5184	0.4272	0.3232	0.2150	0.1120	0.0204

Table 2. y/L values.

e_L/L	0.027	0.053	0.080	0.107	0.133	0.160
e_B/B	y/L					
0.02	0.7819	0.7823	0.7256	0.6634	0.6006	0.5386
0.04	0.3254	0.6211	0.6249	0.583	0.5304	0.4747
0.06	0.0477	0.3894	0.4853	0.4812	0.4469	0.4021
0.08	--	0.1901	0.3303	0.3655	0.354	0.3232
0.10	--	0.0167	0.1902	0.2495	0.2584	0.2419
0.12	--	--	0.0756	0.1439	0.1667	0.1623
0.14	--	--	--	0.0530	0.0833	0.0879
0.16	--	--	--	--	0.0100	0.0204

4. Conclusions

The following conclusions are drawn after the analytical and numerical work done in this study.

- Clear borders among the four zones are established without overlaps. That means borders of the zones given in the technical literature are modified.
- Zone 4 is divided into two. In other words, zone 4 has been modified and a new zone (zone 5) is defined.
- To check whether the resultant load acts in zone 5 (kern), two more criteria have been derived in addition to one in use.
- To calculate the effective areas for the defined five zones, formulas have been derived instead of employing the graphs in calculation of effective areas for all of the cases.
- Since the hand solution of formulas for zone 4, and 5 is impossible, a MATLAB code is generated and the equations are solved. However, any other numerical technique can be employed to solve the equations of (27) and (28).

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