



## Metaheuristic approaches for optimum design of cantilever reinforced concrete retaining walls

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### ABSTRACT

An approach is presented for optimum design of cantilever reinforced concrete (RC) retaining wall via teaching-learning based optimization (TLBO) algorithm. The objective function of the optimization is to minimize total material cost including concrete and reinforcing steel bars of the cantilever retaining wall by considering overturning, sliding and bearing stabilities, bending moment and shear capacities and requirements for design and construction of reinforced concrete structures (TS 500/2000). TLBO algorithm is a simple algorithm without any special algorithm parameters. This innovative approach is providing an advantage to TLBO in terms of easily applying to the problem. The proposed method has been performed on numerical examples and the results are compared with previous approaches. Results show that, the methodology is feasible for obtaining the optimum design of RC cantilever retaining walls.

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### 1. Introduction

The design process of the reinforced concrete (RC) structures involves some decisions, i.e. dimensions of the structural members, material properties (compressive strength of concrete, yield strength of steel), diameter and spacing of bars, etc. done by designer. The security and total cost of the design are closely related with these decisions. Thus, the experience of the designer has an important role in the economy and structural safety. But, it may not enough to find the best design in mean of total cost considering the whole design process of the RC structures containing many design variables and using two materials with extremely different mechanical behavior and unit material cost. For that reason, it must be used or developed methods that are independent of the user experiences in order to ensure best (or optimum) design.

Until recently, the optimum design methods are developed for frames (Balling and Yao, 1997; Guerra and Kiousis, 2006), beams (Barros et al., 2005; Barros et al., 2012; Ferreira et al., 2003), pre-stressed concrete bridges (Sirca and Adeli, 2005), columns (Gil-Martin et al., 2010) and slabs (Ahmadkhanlou and Adeli, 2005).

Despite having successfully applied under specific conditions, the mathematical methods may not present a general methodology for engineering design problems due to the complex (or nonlinear) relationship between design variables. For example, geometry dimension and shape of the cross section of the structural member effects internal forces, displacements and amount (size and spacing) of the bars. Thus, it is not easy to determine whole this relationship with a suitable formulation in order to apply a conventional method and to find optimum results. For that reason, the metaheuristic algorithms are widely used for optimum design of such problems. In the documented methods, the most popular algorithms in the optimum design of RC member are genetic algorithm (Coello et al., 1997; Govindaraj and Ramasamy, 2005; Fedghouche and Tiliouine, 2012; Rafiq and Southcombe, 1998; Rajeev and Krishnamoorthy, 1998; Camp et al., 2003; Lee and Ahn, 2003; Govindaraj and Ramasamy, 2007) and simulated annealing (Paya et al., 2008; Paya-Zaforteza et al., 2009; Ceranic et al., 2001; Yepes et al., 2008; Perea et al., 2008; Rama Mohan Rao and Shyju, 2010). In these studies, several structural members including beams, columns, frames, bridges and plates are handled as design problems.

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In addition to these algorithms, particle swarm optimization (Ahmadi-Nedushan and Varae, 2009) big bang big crunch algorithm (Camp and Akin, 2012; Camp and Huq, 2013; Kaveh and Sabzi, 2012), harmony search algorithm (Kaveh and Abadi, 2011; Akin and Saka, 2010; Akin and Saka, 2012; Bekdaş and Nigdeli, 2012; 2014; Nigdeli et al., 2015), bat algorithm (Bekdaş and Nigdeli, 2016) and teaching-learning based optimization algorithm (Temür and Bekdaş, 2016) are also employed for optimum design of RC members.

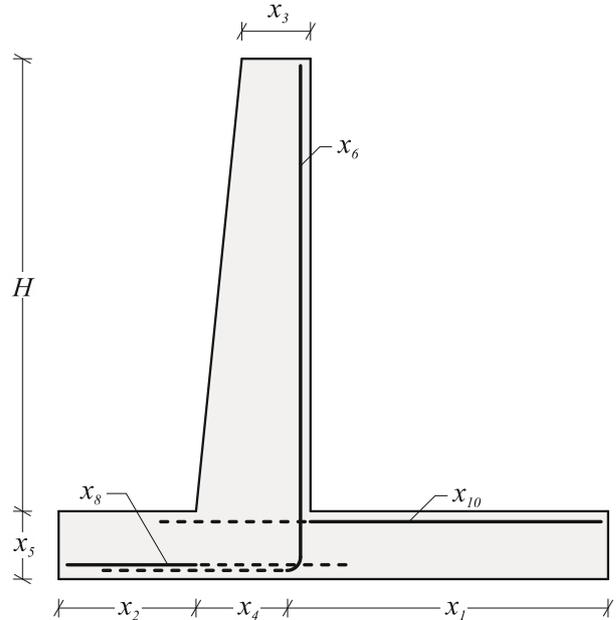
In this paper, a methodology employing teaching-learning based optimization developed by Rao et al. (2011) is presented for optimum design of cantilever retaining RC walls. Turkish Standard Requirements for design and construction of reinforced concrete structures (TS500/2000) regulation are considered in RC design. In order to see the efficiency of the proposed method, the analyses results are compared with the state-of-art algorithms like particle swarm optimization (PSO) and big bang big crunch (BB-BC).

**2. Methodology**

In 2011, Rao et al. proposed a metaheuristic algorithm called teaching-learning based optimization (TLBO) from the inspiration of teaching and learning process in a classroom. Compared with other metaheuristics, one of the innovative parts of the TLBO algorithm is to not use specific algorithm parameters. The optimization process of TLBO algorithm can be summarized in four steps.

**Step I:** In the first step, population number ( $pn$ ), ranges of the design variables and stopping criterion (maximum iteration number) are defined.

**Step II:** Then, the initial solution matrix is constructed by using  $pn$  number of the solution vectors. Each solution vector contains  $vn$  number of randomly generated design variables ( $X_i$ ) which are shown in Fig. 1 and Table 1.



**Fig. 1.** Design variables for cantilever retaining wall.

**Table 1.** Design variables.

	Description	Design variable
Variables related to Cross-section dimension	Heel projection	$X_1$
	Toe projection	$X_2$
	Stem thickness at the top of the wall	$X_3$
	Stem thickness at the bottom of the wall	$X_4$
	Base slab thickness	$X_5$
Variables related to RC design	Diameter of reinforcing bars of stem, $\phi_s$	$X_6$
	Distance between reinforcing bars of stem, $S_s$	$X_7$
	Diameter of reinforcing bars of the toe, $\phi_t$	$X_8$
	Distance between reinforcing bars of the toe, $S_t$	$X_9$
	Diameter of reinforcing bars of the heel, $\phi_h$	$X_{10}$
	Distance between reinforcing bars of the heel, $S_h$	$X_{11}$

These variables (possible design solutions) are randomized (Eq. (1)) within a defined range using upper ( $X_i^{max}$ ) and lower limits ( $X_i^{min}$ ).

$$X_i^{min} \leq X_i \leq X_i^{max} . \tag{1}$$

By positioning each solution vector to a row, general form of the solution matrix can written as

$$CL = \begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,vn} \\ X_{2,1} & X_{2,2} & \dots & X_{2,vn} \\ \vdots & \vdots & & \vdots \\ X_{pn-1,1} & X_{pn-1,2} & \dots & X_{pn-1,vn} \\ X_{pn,1} & X_{pn,2} & & X_{pn,vn} \end{bmatrix} . \tag{2}$$

Then, strength capacity and safety for stability of each retaining walls are checked by using constrains given in Table 2. The requirements of TS500/2000 regulation are considered for calculation strength capacity of sections and extremum limits.

Before the next step, the objective functions (total costs) for each retaining walls are calculated (Eq. 3) and stored in a vector for future comparisons.

$$\min f(X) = C_c \cdot V_c + C_s \cdot W_s. \quad (3)$$

In Eq. (3),  $C_c$  is unit cost of concrete  $C_s$  unit cost of steel,  $V_c$  is volume of concrete and  $W_s$  is weight of steel per unit length.

**Step III:** According to TLBO rules, in the third step, teacher and learner phases are respectively applied in order to improve solutions. Mathematically, teacher ( $tp$ ) and learner ( $lp$ ) phases can be written as

$$X_{new,i}^{ip} = X_{old,i} + rnd(0,1) \cdot (X_{teacher} - T_F \cdot X_{mean}), \quad (4)$$

$$X_{new,i}^{lp} = \begin{cases} X_{old,i} + rnd \cdot (X_i - X_j); & f(X_i) > f(X_j) \\ X_{old,i} + rnd \cdot (X_j - X_i); & f(X_i) < f(X_j) \end{cases}, \quad (5)$$

respectively. In the Eqs. (3) and (4),  $X_{teacher}$  is the vector with best (minimum total cost) objective function in the solution matrix and it is defined as

$$X_{teacher} = x_{\min f(X)}. \quad (6)$$

$X_{mean}$  is the mean value of the design variables formulated as

$$X_{mean} = \frac{\sum_{i=1}^{pn} X_i}{pn}. \quad (7)$$

$T_F$  is an integer number called teaching factor written as

$$T_F = round [1 + rnd(0.1)] \rightarrow \{1 - 2\}, \quad (8)$$

and it can be 1 or 2 according to the  $rnd$  (random reel number between 0 and 1) value.  $X_{old,i}$  and  $X_{new,i}$  represent old and new values of the variables, respectively. After updating the design variables at each phase, the objective function of the new vector is calculated and compared with the values of the old vector.

**Step IV:** In this step, the stopping criterion is checked. The iterative process continue from the Step III, until the stopping criterion is satisfied.

**Table 2.** Constraints on strength and dimensions of wall.

Description	Constraints
Safety for overturning stability	$g_1(X): SF_{O,design} \geq SF_0$
Safety for sliding stability	$g_2(X): SF_{S,design} \geq SF_5$
Safety for bearing capacity	$g_3(X): SF_{B,design} \geq SF_B$
Minimum bearing stress, $q_{min}$	$g_4(X): q_{min} \geq 0$
Flexural strength capacities of critical sections, $M_d$	$g_{5-7}(X): M_d \geq M_u$
Shear strength capacities of critical sections, $V_d$	$g_{8-10}(X): V_d \geq V_u$
Minimum reinforcement areas of critical sections, $A_{smin}$	$g_{11-13}(X): A_s \geq A_{smin}$
Maximum reinforcement areas of critical sections, $A_{smax}$	$g_{14-16}(X): A_s \leq A_{smax}$
Maximum steel bars spacing of critical sections, $S_{max}$	$g_{17-19}(X): S \leq S_{max}$
Minimum steel bars spacing of critical sections, $S_{min}$	$g_{20-22}(X): S \geq S_{min}$
Minimum concrete cover, $c_c$	$g_{23}(X): c_c \geq 70 \text{ mm}$
Sectional limits	$g_{24}(X): (X_2 + X_3) \geq X_1$
	$g_{25}(X): (X_6 + X_7) \geq X_1$
Reinforcement development lengths, $l_{db}$ and hook lengths, $l_{dh}$	$g_{26}(X): l_{db,stem} \geq (X_5 - c_c)$ or $l_{dh,stem} \geq (X_5 - c_c)$
	$g_{27}(X): l_{db,toe} \geq (X_1 - X_2 - c_c)$ or $12d_{b,toe} \geq (X_5 - c_c)$
	$g_{28}(X): l_{db,heel} \geq (X_2 + X_3 - c_c)$ or $12d_{b,heel} \geq (X_5 - c_c)$
	$g_{29}(X): l_{db,key} \geq (X_5 - c_c)$ or $l_{dh,key} \geq (X_5 - c_c)$

### 3. Numerical Example

The proposed methodology is applied to a cantilever retaining wall benchmark problem that described in Saribaş and Erbatur (1996). Design constraints and ranges of design variables for the problem can be seen in Table 3. The optimum costs were investigated under three different cases related with backfill slope angle, surcharge load and compressive strength of concrete. In order to compare the effectiveness of the presented approach, two other metaheuristic methods; PSO (Ahmadi-Nedushan and Varraee, 2009) and BBBC (Camp and Akin, 2012) were also adapted to the numerical example.

The convergence to the optimum cost value of the methods can be seen in Fig. 1. As seen from the plot, although all methods are achieved to find the optimum result, the TLBO is better than the other methods. As a result, the TLBO is the best method in point of computational effort (for obtaining the optimum results more quickly).

In order to investigate statistical treatment of results in comparisons, averages and standard deviations of methods were calculated for different values of backfill slope angle. For each value of the backfill slope angle 100 independent runs were performed. The averages and standard deviations can be seen in Figs. 2 and 3.

**Table 3.** Design constants and ranges of design variables.

Definition	Symbol	Unit	Value
Height of stem	$H$	m	3.0
Yield strength of steel	$f_y$	MPa	420
Compressive strength of concrete	$f'_c$	MPa	30
Concrete cover	$c_c$	mm	70
Max. aggregate diameter	$D_{max}$	mm	16
Elasticity modulus of steel	$E_s$	GPa	200
Specific gravity of steel	$\gamma_s$	t/m <sup>3</sup>	7.85
Specific gravity of concrete	$\gamma_c$	kN/m <sup>3</sup>	23.5
Cost of concrete per m <sup>3</sup>	$C_c$	₺	119
Cost of steel per ton	$C_s$	₺	1751
Design load factor		LF	1.7
Surcharge load	$q$	kPa	20
Backfill slope angle	$\beta$	°	10
Internal friction angle of retained soil	$\phi_R$	°	30
Internal friction angle of base soil	$\phi_B$	°	0
Unit weight of retained soil	$\gamma_R$	kN/m <sup>3</sup>	17.5
Unit weight of base soil	$\gamma_B$	kN/m <sup>3</sup>	18.5
Cohesion of retained soil	$c_R$	kPa	0
Cohesion of base soil	$c_B$	kPa	125
Depth of the soil in front of wall	$D$	m	0.5
Safety for overturning stability	$SF_{O,design}$	-	1.5
Safety for sliding stability	$SF_{S,design}$	-	1.5
Safety for bearing capacity	$SF_{B,design}$	-	3.0
Range of stem thickness at top	$h_{stent}$	m	0.2-3
Range of heel projection	$h_{basew}$	m	0.2-10
Range of toe projection	$h_{toepro}$	m	0.2-10
Range of stem thickness at the bottom of wall	$h_{stemb}$	m	0.2-3
Range of base slab thickness	$h_{baseslab}$	m	0.2-3
Range of diameter of reinforcing bars of stem	$\phi_s$	mm	16-50
Range of diameter of reinforcing bars of toe,	$\phi_t$	mm	16-50
Range of diameter of reinforcing bars of heel	$\phi_h$	mm	16-50

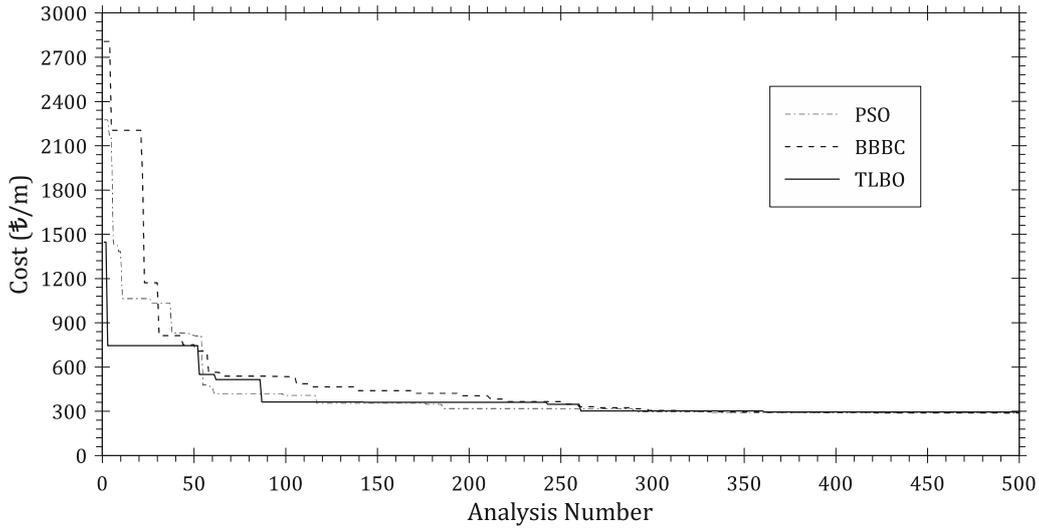


Fig. 1. Convergence to optimum results of methods.

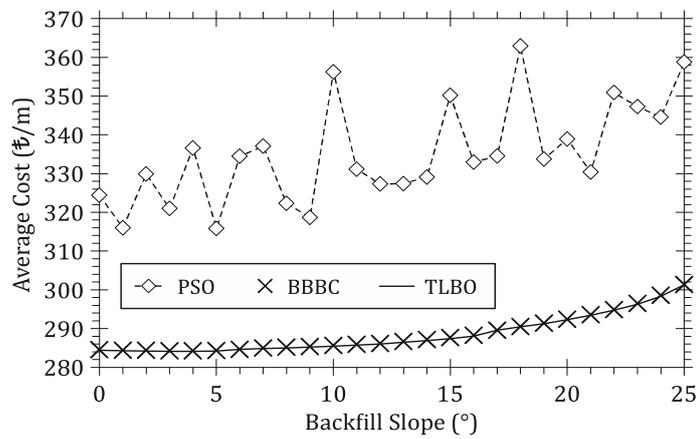


Fig. 2. Average cost values of the methods for different values of backfill slope angle.

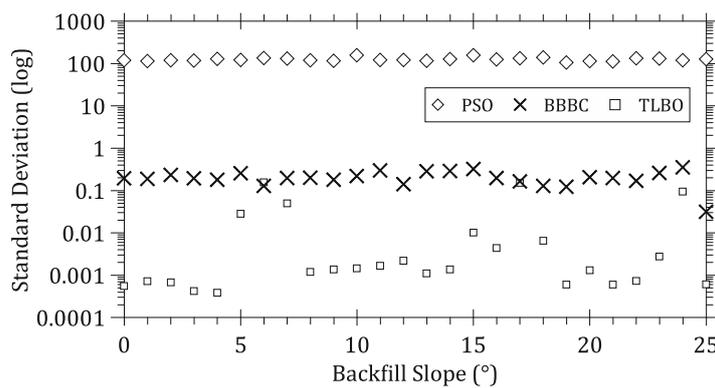
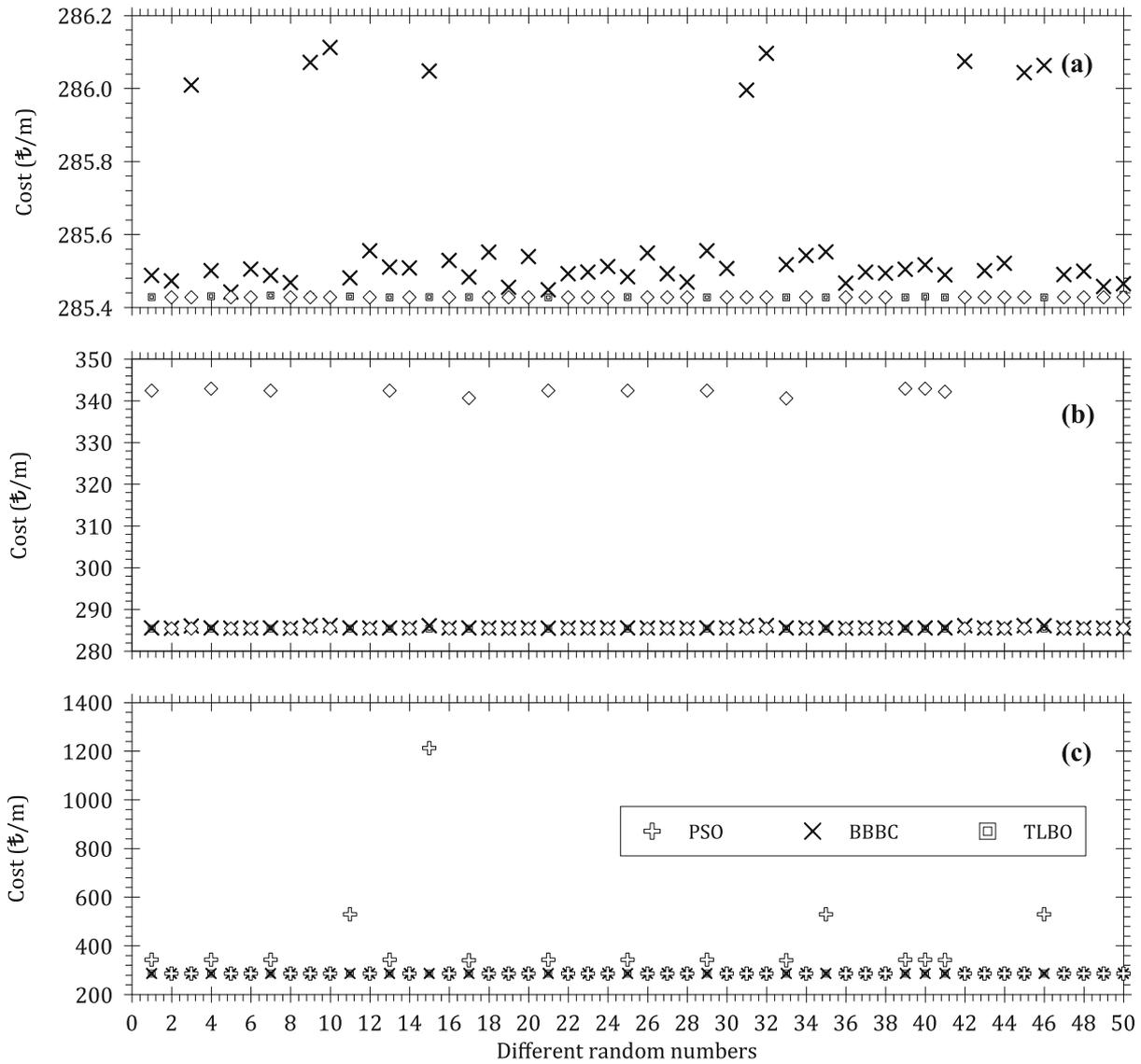


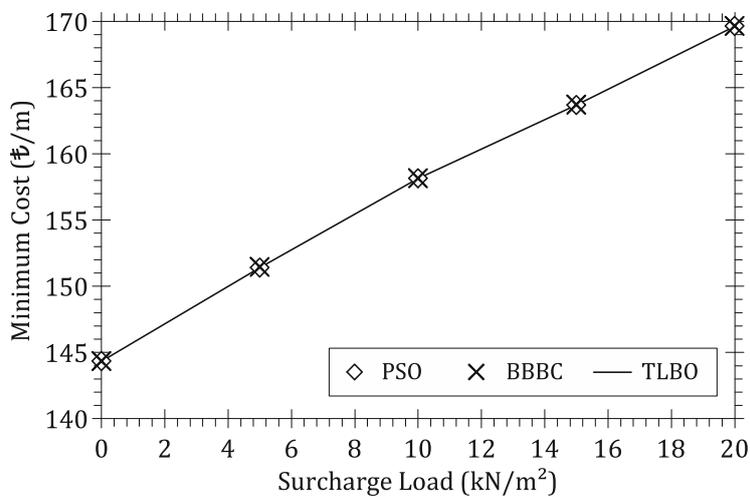
Fig. 3. Standard deviation values of 100 independent runs for different values of backfill slope angle.

100 independent runs were also conducted with different random numbers for investigation of the optimum results sensitivity of the methods. As seen from the Fig. 4, although the optimum results are obtained approximately 3 times bigger than true optimum value for some analyses of PSO and BBBC approaches, true optimums in all cases of TLBO method are found. According to the results, the algorithms can be sorted as TLBO, BBBC and PSO from the best to worst one.

In Figs. 5 and 6, effects of surcharge load and compressive strength of concrete to optimum cost can be seen. Minimum cost value is changed between 145€-170€ and 270€-310€ for different surcharge loads and compressive strengths respectively. Also, an approximate linear relationship is observed for both cases.



**Fig. 4.** Optimum cost distribution plot for 100 independent designs: (a) A more detailed graph of the 285.4-286.2 cost range; (b) A more detailed graph of the 280-350 cost range; (c) All solutions.



**Fig. 5.** Minimum cost vs. surcharge load plot.

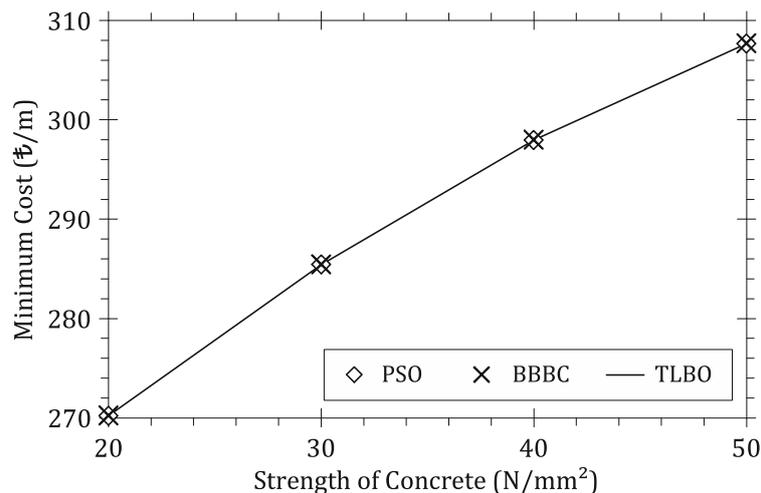


Fig. 6. Minimum cost vs. strength of concrete plot.

#### 4. Conclusions

The optimum cost for cantilever retaining RC walls were investigated for different conditions such as, back-fill slope angle, surcharge loads and compressive strength of concrete. In addition to proposed method with TLBO metaheuristic approach, optimization process were also conducted PSO and BBBS algorithms to show effectiveness of the presented approach. By conducting 100 independent run, statistical treatment of results were observed for all algorithms. According to the analyses results, all algorithms are successful in finding optimum design of the wall for all cases. But, the best computational time for optimum results is obtained for TLBO algorithm. Additionally, sensitivity of the TLBO is better than PSO and BBBC algorithm. As conclusion, TLBO is effective and suitable approach for optimum design of cantilever retaining RC walls considering TS 500/2000 regulation.

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